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AIR ENGINES

by H. RINIA and F. K. DU PRÉ. 621.412:536.8

Around the middle of the last century considerable interest was shown for a time in the air engine, but this was quickly supplanted by the development of internal combustion engines. This was not due to the thermodynamic process involved, but to the technical constructions possible in those times. Recent research work in Philips laboratories has now shown that very satisfactory results can be attained with air engines, if they are built in accordance with modern conceptions in regard to heat transfer, flow resistance, properties of materials, etc. In this article the theoretical principles of the air process are discussed.

Introduction

When reviewing the processes that have been applied in the course of years in an attempt to convert heat into mechanical work, it is remarkable that the air process at present in disuse was considered so very promising for quite a while. It was first used in 1817 by Stirling, and afterwards various air engines, some of them very large, were built and put into practical use. Some people were even of the opinion that the air engine was a serious competitor of the steam engine, that had previously come into use.

These early air engines, however, were so unwieldy, slow and uneconomical that it is not surprising, that they were entirely supplanted after the invention of the internal combustion engine. After a short while, therefore, interest in the air process faded out almost entirely.

Now it was known that according to thermodynamics the air process was to be considered as one of the most economical methods of generating mechanical energy from heat. In the previous century, however, it was technically impossible to construct a good air engine. As a result of the extensive theoretical and experimental research work that has been carried out on this subject for several years past in the Philips laboratories, it has been established that present-day technology is indeed capable of exploiting the theoretical possibilities of the air process efficiently.

By making use of modern materials and modern conceptions of heat-transfer and flow-resistance it was found possible to apply the air process in engines capable of performing 3000 r.p.m, with most satisfactory figures for weight and efficiency. In this article we shall confine ourselves to the main theoretical factors which play a part in the air process.

Principle of the air engine

We shall explain the principle of the air engine with the help of a very much simplified model. Let us imagine a cylinder divided into two parts in open connection with each other. One part, the so-called hot space, is kept at a high temperature T_h by means of a heater, while the other part, the cold space, is kept at a low temperature T_c by a cooler. In each of the two parts of the cylinder is a moving piston. A certain quantity of air is enclosed between the two pistons. We shall assume for the present that the transfer of heat between the cylinder and the air in it is so good that the air in the hot space is always at the temperature T_h and that in the cold space always at the temperature T_c.

We now cause the air in the cylinder to pass through a cycle consisting of four phases. The four

positions of the pistons at the moments when one phase passes over into the next are shown in *fig. 1a* as *I* to *IV*. The cycle is as follows:

At position *I* all the air is in the cold space, which has the maximum volume V_1.

The transition *I-II* takes place isothermally, according to our assumption about heat transfer; the air is compressed to the volume V_2.

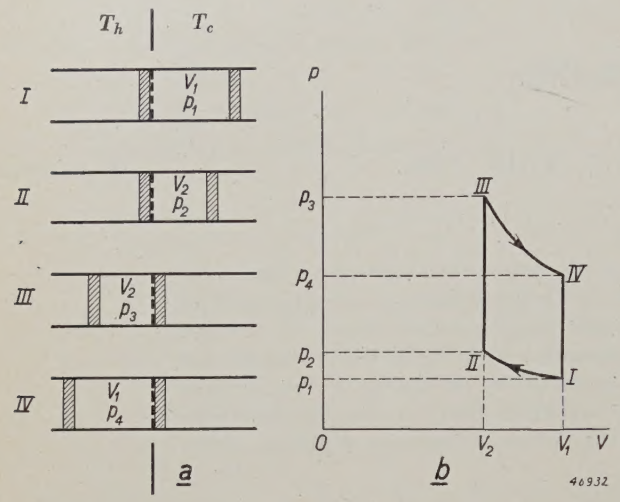

Fig. 1. Diagram explaining the action of an air engine.

a) The engine consists of a cylinder which is divided into a hot and a cold part with temperatures of T_h and T_c respectively, which are in open connection with each other. In both parts there is a piston. Between the two pistons is a definite quantity of air. In the engine, a cyclic process takes place four stages of which are indicated:
The transition *I-II* takes place isothermally at T_c.
The transition *II-III* takes place isochorically, *i.e.* with constant volume.
The transition *III-IV* takes place isothermally at T_h.
The transition *IV-I* takes place isochorically.

b) The *p-V* diagram of the cycle through which the work medium passes. Owing to the fact that during the hot expansion (*III-IV*) the pressure is higher than during the cold compression (*I-II*), the work produced during the expansion is larger than the work supplied during the compression. As a result there is a positive work surplus for the cycle as a whole. This surplus is represented by the area of the curvilinear quadrilateral *I-II-III-IV*.

At position *II* all the air is still in the cold space, which, however, now has the minimum volume V_2.

The transition *II-III* takes place at constant (minimum) volume. The air is displaced from the cold to the hot space.

At position *III* all the air is in the hot space, which has the minimum volume V_2.

The transition *III-IV* again takes place isothermally; the air expands from the minimum volume V_2 to the maximum volume V_1.

At position *IV* all the air is in the hot space, but this now has the maximum volume V_1.

The transition *IV-I* again takes place at constant (maximum) volume, the air being again displaced to the cold space.

It is not difficult to see that the process outlined can in principle be applied for an engine.

In the transitions *II-III* and *IV-I* no mechanical work at all is done. The work which has to be supplied to the one piston is exactly the same as the work done by the other (all mechanical friction being disregarded of course), because the volume of the air remains constant during these transitions. Therefore we need only consider the transitions *I-II* and *III-IV*. The first is a compression and thus work must be done. This compression, however, takes place at a low temperature and therefore also at a low pressure. The second is an expansion and thus produces work. Since this takes place at a high temperature (and because the same series of volume changes takes place as in the compression, although in reverse order) the average pressure is higher than in the compression and the work produced is greater than the work required for the compression. As a result there is a surplus of work per complete cycle.

The action of engines with one piston per cylinder usually consists in the piston being displaced during one half of a revolution under the influence of a high pressure and then moving back again during the succeeding half revolution against a lower pressure. In the presently described construction of the air engine this process is divided between two pistons, and between the two processes there is each time a workless phase. Otherwise the situation is quite analogous. In *fig. 1b* the *p-V* diagram is drawn for the cycle outlined; the phases described above can easily be recognized. The work gained per cycle is represented in the diagram by the area of the curvilinear quadrilateral *I-II-III-IV*.

From the foregoing it is evident that the surplus of work is obtained owing to the fact, that the isothermal expansion of the air takes place at a high temperature and the isothermal compression at a low temperature. This is one of the fundamental principles of the air engine, and further use will be made of it in the following.

The cyclic process as described above is of course in that form very difficult of realization technically; the four-phase piston movements would lead to very complicated constructions. It can, however, be so modified, while still retaining the principle, as to make it technically possible of achievement. Let us now imagine the two pistons as being coupled by a combination of driving rods in such a way that the movements of the hot piston are followed, with a

certain phase difference, by those of the cold piston. In *fig. 2* a very schematic representation is given of how that can be done. With this form of construction it is no longer possible to distinguish the separate phases as they begin to overlap somewhat, and in particular the expansion does not take place completely in the hot space nor the compression completely in the cold space. However, when the

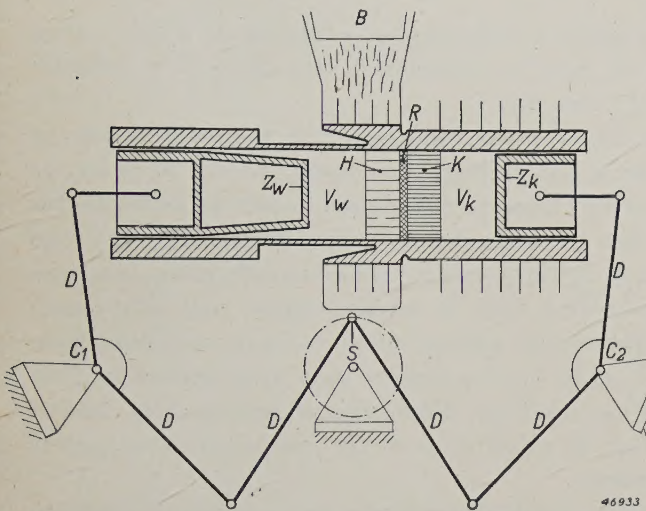

Fig. 2. Diagram of the coupling of the two pistons of the air engine. V_w hot space, V_k cold space, Z_w hot piston, Z_k cold piston, D driving mechanism, C_1 and C_2 fixed pivots, S crank shaft, B burner. In the figure the heater H, regenerator R and cooler K, which will be discussed later, are also indicated.

phase difference is suitably chosen, *i.e.* in the neighbourhood of 90°, it can be shown that the expansion does take place mainly in the hot space and the compression mainly in the cold space, so that this new cyclic process will also furnish a surplus of work per cycle, *i.e.* in this case per revolution of the crank shaft.

Variation of the air pressure p in the engine during one cycle

Before we proceed to calculate the work gained per revolution, we shall first determine how the air pressure in the engine varies, on the assumption that the engine runs uniformly with a circular frequency ω. This can always be accomplished by fitting a flywheel with a sufficiently large moment of inertia on the shaft of the engine. We will assume further, that the volumes of the hot and cold spaces then vary purely sinusoidally. The following notation will be used:

V_0 maximum volume of the hot space,
V_h volume of the hot space at any arbitrary moment,
v ratio of the maximum volumes of cold and hot spaces,
vV_0 maximum volume of the cold space,
V_c volume of the cold space at any arbitrary moment.

Finally we assume that V_h and V_c have zero as lower limit.

For V_h and V_c we may then write

$$V_h = \tfrac{1}{2}\, V_0\,(1 + \cos \omega t)\,, \quad . \quad . \quad . \quad . \quad (1)$$

$$V_k = \tfrac{1}{2}\, vV_0 \left\{ 1 + \cos\,(\omega t - \varphi) \right\}. \quad (2)$$

φ being the phase difference.

Having regard to the practical applications we also assume that the engine contains a certain dead space V_s, which we shall define as that part of the total space in the engine available for air which takes no part in the movements. We assume that the air in that space has an average constant temperature of T_s.

The variation of pressure as a function of time now follows from the condition that the mass of the work medium, in this case air, must be constant. We shall calculate this mass on the assumption that air is a perfect gas. In the hot, cold and dead space there is therefore:

$$\frac{M}{R}\cdot\frac{pV_h}{T_h}\ \text{gram},\ \frac{M}{R}\cdot\frac{pV_c}{T_c}\ \text{gram and}\ \frac{M}{R}\cdot\frac{pV_s}{T_s}\ \text{gram}.$$

In this expression p is the pressure, M the average molecular weight of air and R the gas constant. The constancy of the mass of the work medium is now expressed by the equation:

$$\frac{M}{R}\cdot\frac{pV_h}{T_h} + \frac{M}{R}\cdot\frac{pV_h}{T_c} + \frac{M}{R}\cdot\frac{pV_s}{T_s} = C'.$$

We now replace the right-hand member by $M/R \cdot CV_0/2T_c$, where C is a constant. This method of notation simplifies the calculation.

In the expression obtained we now substitute the expressions for V_h and V_c.

After slight reduction this gives:

$$\frac{C}{2p} = \frac{(1+\cos\omega t)\ T_c}{2T_h} + \frac{v\left\{1+\cos\,(\omega t-\varphi)\right\}\,T_c}{2T_c} + \\ + \frac{V_s}{V_0}\cdot\frac{T_c}{T_s}.$$

We now introduce the following quantities:
τ, the temperature ratio: $\tau = T_c/T_h$;
s, the reduced dead space: $s = V_s/V_0 \cdot T_c/T_s$.
(In practice for example $T_h = 875°$ K and $T_c = 350°$ K, so that $\tau = 0.4$; s is usually about 0.5).
The equation thus becomes:

$$\frac{C}{p} = \tau\cos\omega t + v\cos\,(\omega t - \varphi) + \tau + v + 2s\,,\ \text{or}$$

$$\frac{C}{p} = (\tau + v\cos\varphi)\cos\omega t + v\sin\varphi\sin\omega t + \tau + \\ + v + 2s\,.$$

For this we may now write:

$$\frac{C}{p} = \sqrt{\tau^2 + v^2 + 2\,\tau\,v\cos\varphi}\cos(\omega t - \Theta) + \tau + v + 2s,$$

where Θ is determined by:

$$\operatorname{tg}\Theta = \frac{v\sin\varphi}{\tau + v\cos\varphi}.$$

If we now introduce the abbreviations:

$$\sqrt{\tau^2 + v^2 + 2\,\tau\,v\cos\varphi} = A\,;$$

$$\tau + v + 2s = B \text{ and } \frac{A}{B} = \delta,$$

we find for p:

$$p = \frac{C}{B}\,\frac{1}{1 + \delta\cos(\omega t - \Theta)}.$$

From this it follows, that the maximum value of p is determined by:

$$p_{\max} = \frac{C}{B}\,\frac{1}{1-\delta}.$$

With the help of this last expression we may write for p:

$$p = p_{\max}\frac{1-\delta}{1 + \delta\cos(\omega t - \Theta)}. \quad . \quad . \quad (3)$$

The variation of pressure found can be represented very simply in polar coordinates. Equation (3) is the equation of an ellipse with a focus at the origin, when p is chosen as radius vector and ωt as vectorial angle. This ellipse is shown in *fig. 3.*

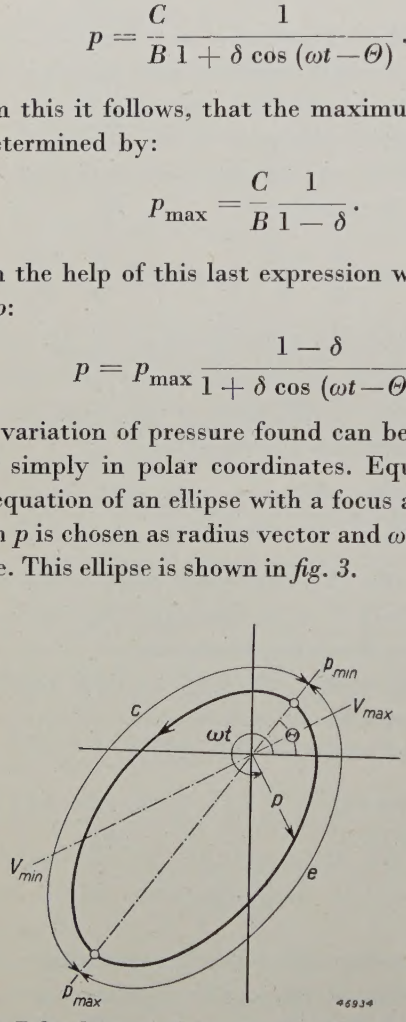

Fig. 3. Polar diagram of the variation of the pressure p in the engine (with sinusoidally moving pistons with a constant phase difference) as a function of the time t. The relation is represented as an ellipse with one focus at the origin. The vertices of this ellipse thus represent the highest and the lowest pressures, $p_{\max}$ and $p_{\min}$. The points corresponding to the positions where the total volume ($V = V_h + V_c + V_s$) is a maximum or a minimum lie on a line through the origin. The slope of this line with respect to the horizontal axis is smaller than the corresponding slope Θ of the axis of the ellipse. Thus the average pressure during the transition from $V_{\min}$ to $V_{\max}$ is higher than during the transition from $V_{\max}$ to $V_{\min}$. Arc e represents the expansion, arc c the compression.

The minimum and maximum pressures are reached when $\omega t = \Theta$ and $\Theta + 180°$, respectively. These values are represented by the vertices of the ellipse, which therefore lie on a line through the origin making an angle Θ with the zero line. In the figure it is also indicated for what value of ωt the whole work volume,

$$V = V_h + V_c + V_s$$

reaches a maximum or a minimum. These values can easily be determined with the help of formulae (1) and (2).

The corresponding points in the diagram also lie on a line through the origin, which, however, as follows from the calculation, makes an angle smaller than Θ with the zero line, provided $\varphi > 0$ and $\tau < 1$. This condition, as we already know, is always complied with in an air engine, and as a consequence the average pressure upon transition from $V_{\min}$ to $V_{\max}$ is greater than upon transition from $V_{\max}$ to $V_{\min}$. From this it immediately follows that in a full cycle of the engine positive work is gained.

The result obtained may be summed up as follows. An installation of two cylinders in open connection with each other, each closed by a piston with a closed quantity of air between the two pistons, one cylinder being kept at a constant high temperature and the other at a constant low temperature, acts as an air engine as soon as the pistons are made to move sinusoidally with a constant phase difference, with the variations in the volume of the hot cylinder preceding those in the cold cylinder.

Power produced

When a work medium goes through a cyclic process the work gained is in general given by the expression:

$$\oint p\,\mathrm{d}V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This quantity can be interpreted geometrically as the area of the closed curve which represents the cycle in the p-V diagram.

In our case $V = V_h + V_c + V_s$, and since V_s is constant, we may also write for the expression above :

$$\oint p\,\mathrm{d}V_h + \oint p\,\mathrm{d}V_c.$$

Each of these integrals may in the same way be conceived as the area of a closed curve in the p-V diagram. We must then plot, not the corresponding values of p and V, but those of p and V_h and of p and V_c, respectively.

The latter two integrals can also be interpreted physically, namely as the work performed separately by the hot and cold pistons respectively.

Formula (3), which represents the variation in pressure, in combination with formulae (1) and (2) for the variations of V_h and V_c, now enables us to construct the three p-V diagrams and to calculate the quantity of the work gained per revolution.

The three p-V diagrams are shown in *fig. 4* for the case that $V_0 = 2300$ cm³, $v = 1$ and $p_{max} = 40$ kg/cm².

the work per revolution by the number of revolutions per second $\omega/2\pi$, and we therefore obtain:

$$N = \frac{\omega}{2\pi} \oint p\, \mathrm{d}V = \frac{\omega}{2\pi} \oint p\, (\mathrm{d}V_h + \mathrm{d}V_c)\,.$$

If we substitute in this expression the values found for p, V_h and V_c (given by (1), (2) and (3) we obtain after simplification:

$$N = \frac{\omega}{2} p_{max} V_0\, v\, (1-\tau)\, \frac{\sin\varphi}{A}\, \frac{1-\delta}{\delta} \left(\frac{1}{\sqrt{1-\delta^2}} - 1\right) \quad . \; . \; . \; (5)$$

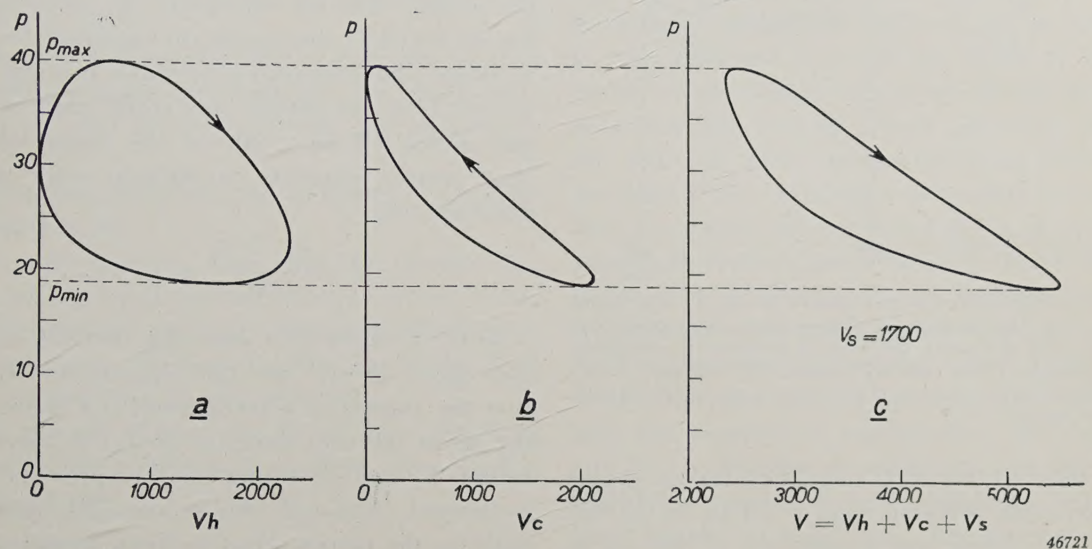

Fig. 4. p-V diagrams for an air engine with two pistons moving sinusoidally with a constant phase difference.
a) The diagram for the hot space (volume V_h).
b) That for the cold space (volume V_c).
c) The diagram for the whole working volume, (volume V, dead space, V_s).
$(V_h)_{max} = (V_c)_{max} = 2300$ cm³; $p_{max} = 40$ kg/cm², $\varphi = 90°$.
From the cyclic direction in the diagrams it may be seen that the hot piston produces work per revolution, that the cold piston consumes work in that time interval and that there is a total positive surplus of work per revolution.

Fig. 4*a* represents the diagram for the hot air, fig. 4*b* that for the cold air, fig. 4*c* finally giving the diagram for the whole working volume. From the cycle in the diagrams it can be seen that in case *a* positive work is done, and in case *b* negative work, while in case *c* (which is the sum of the amounts of work in *a* and *b*) it is again positive.

From the formulae given it is easily deduced that the surplus of work is only positive if the variations in the volume of the hot space are ahead in phase of those of the cold space. If this phase difference (the quantity φ in our calculations) were chosen equal to zero, the p-V diagrams would shrink to sections of curves and no work would be gained. If φ becomes negative the engine consumes work; this is applied in refrigerators, which are mentioned at the end of this article.

The power N, *i.e.* the work produced by the working medium per second, is obtained by multiplying

If all quantities are expressed in cgs units we of course obtain N in ergs/sec. By division by 7.36×10^9 N is obtained in the more customary unit HP. It has been found that the quantity N calculated in this manner gives a good approximation of the power actually produced by the air process. Of course this does not mean that the amount N thus obtained represents the effective power of the engine, because a not inconsiderable percentage is always consumed in the engine itself as a result of friction of the pistons and the other moving parts and from several other causes. In the case of small engines this may be around 25%, whilst in larger multi-cylinder engines it is less [1]).

[1]) Considerations about the variations of pressure and power analogous to those given above can be found in publications of Schmidt of 1862 and 1871, which refer to the models of air engines then known.

In *fig. 5* the power values calculated according to (5) are plotted as a function of the phase angle φ. (It must be kept in mind that this expression contains φ not only because the factor sin φ occurs in it, but because δ and A are also functions of φ.)

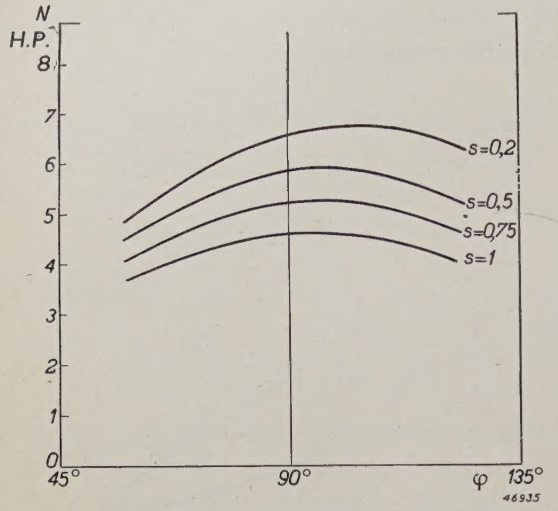

Fig. 5. Relation between the power N of an air engine (calculated on the assumption of isothermal compression and expansion) and the phase difference φ between the variations of the volumes of hot and cold space, for different values of the reduced dead space s. $V_0 = 1$ litre, $p_{max} = 10$ kg/cm^2, $n = 1000$ r.p.m., $v = 1$.

The diagram has been calculated for an engine with maximum pressure $p_{max} = 10$ kg/cm^2, a hot volume $V_0 = 1$ litre and a speed of $n = 1000$ r.p.m. It is further assumed, as is always approximately true in practice, that the hot and cold volumes are equal, thus $v = 1$. Since the power is proportional to p_{max}, V_0 and ω, the graph can also be used directly for engines with different values of those three parameters.

This diagram shows that the optimum value of the phase angle lies between 90° and 110°, while it is important not to choose the reduced dead space too large.

Regulation of the power produced

The formulae derived also enable us to realize how the power of air engines can be regulated. In the case of an engine of given construction and dimensions the maximum pressure occurring depends upon the amount of air taking part in the process. By varying that amount the maximum pressure can therefore also be varied. Now it is found from formulae (3) that when p_{max} is varied the pressure p at every moment changes in the same ratio and thus also the integral $\oint p\, dV$ and with that the power. From this it follows, that the power of the air engine can be varied by varying the amount of air taking part in the process.

The necessary heat Q and the thermodynamic efficiency

We shall now pass on to a discussion of the thermodynamic efficiency η of the air engine and, in doing so, take the opportunity to draw attention to several points of essential importance in the construction of such an engine.

We define the efficiency in the usual way as:

$$\eta = \frac{N}{Q},$$

where N is the power and Q the heat necessary per second. Here we will quote the following well-known Carnot theorem of thermodynamics: When a system passes through a reversible process, taking up heat from the outside only at the temperature T_h and giving off heat only at the temperature T_c, that process produces mechanical work with an efficiency of:

$$\eta = \frac{T_h - T_c}{T_h} = 1 - \tau. \quad . \quad . \quad . \quad . \quad (6)$$

So far we have been assuming that the air in the hot space always has the temperature T_h, thus that the expansion is isothermal, and likewise that the air in the cold space is always at the temperature T_c and thus that the compression is also isothermal. Although this is not quite correct we shall for the present keep to these assumptions, so that then we may say that the efficiency of the engine would be represented by the above formulae as soon as the process becomes reversible. In the form, in which we have so far been discussing the engine, however, that will certainly not be the case. When, as is assumed in fig. 1, the air is pressed at a constant volume (isochoric) from the cold to the hot space (*II-III*) and then returned (*IV-I*), we are dealing with irreversible processes which are accompanied by loss of energy. As a result of these processes a certain amount of heat is transferred per revolution from the temperature T_h to T_c, which produces no work at all. Because of this the value of Q will be larger than in the theoretical case and η correspondingly smaller. What has been observed here for the schematic case of fig. 1 holds equally so for the engine of fig. 2, which more nearly approaches the practical case.

It is, however, possible to cause the transitions from the hot to the cold space and *vice versa* to take place reversibly, at least theoretically. For that purpose a so-called regenerator is introduced between the two spaces, in which the temperature of the air changes gradually from T_h to T_c as it flows through. In this way the exchange

of heat always takes place between bodies (namely the air and the respective part of the regenerator) which differ only very little in temperature. Thus the heat given off by the air remains stored in the regenerator and as the air flows back after half a cycle this heat is again used to bring the temperature of the air up to T_h. It is nowadays possible to make regenerators having an efficiency of 95% and more; by efficiency in this case is meant the percentage of the heat contained in the air inflow that is stored in the regenerator and given back to the air as it returns. The part of the heat, not stored, is carried off by the cooler and thus lost for the cycle.

If we now assume that the regenerator used is ideal and, moreover, disregard the flow resistances, we may say that the cyclic process applied is reversible.

In that case, therefore, the efficiency is given by the equation (6).

It is interesting to note that the process discussed, which clearly differs from the Carnot cycle, has, nevertheless, the same efficiency. We further call attention to the fact that the size of the dead space has no effect on the efficiency.

With the aid of the value found for η, the heat to be supplied to the work medium per second is determined by:

$$Q = \frac{N}{1-\tau}.$$

The heat to be dissipated is:

$$Q - N = \tau\, Q = \frac{\tau\, N}{1-\tau}.$$

This is the theoretically lowest possible amount. If the process is not reversible the heat to be dissipated is greater and the mechanical work and thereby the efficiency smaller.

We have already pointed out that the efficiency of the engine will be considerably lower than the efficiency of the process taking place in the engine, since owing to mechanical frictions the engine itself consumes an appreciable percentage of the power N furnished, while, moreover, the supply of heat always involves certain losses.

The internal process, however, even in the case of the engines already constructed, reasonably satisfies the equation $\eta = 1-\tau$, so that the internal efficiency can only be affected by the choice of the temperature of the hot and the cold spaces. In order to obtain a high internal efficiency it is important to provide that $\tau = T_c/T_h$ is as small as possible, which in practice means that T_h should be as high as possible and T_c as low as possible. For practical reasons, however, T_h is not chosen higher than 900 to 1000° K. T_c is usually around 300 to 350° K. Since the pressure does not occur in the expression for the efficiency it can be chosen quite independently, being dependent only on the dimensions of the engine and the properties of the materials used.

Until now we have expressly assumed that the air in the hot space is always at the temperature T_h and that in the cold space always at T_c. In order to accomplish this as nearly as possible it is not sufficient to keep the walls of the two spaces at the temperatures prescribed, transfer of heat from the wall to the air not being sufficient for that. A good air engine must therefore be provided with a specially constructed heater and also a specially constructed cooler. (In their construction provision has to be made on the one hand for an efficient and rapid heat transfer to the air, while on the other hand the flow resistance may not be too large, since it consumes power.) The heater and cooler are placed between the hot space and the regenerator and between the cold space and the regenerator, respectively (see also fig. 2), and their construction, to which we shall return in a later article, is such that after passing through them the whole volume of air is at the temperature T_h or T_c respectively. This arrangement guarantees that the air always flows into the hot space at the temperature T_h and into the cold space at T_c.

This, however, does not ensure that the air also remains continuously at those temperatures. Let us consider for example the expansion in the hot space. In a high-speed engine the expansion takes place within a fraction of a second and consequently the parts of the air at some distance from the wall will not receive the necessary heat quickly enough and will therefore drop in temperature. (Imagine here, for example, that the heater is a ring surrounding the hot space, so that heat may be fed to the work medium also through the wall of the hot space.) These parts of the air will not expand isothermally, but approximately adiabatically. Similarly for the compression in the cold space, the air there will assume a temperature higher than T_c. These adiabatic processes mean that the whole process is no longer reversible. At the end of the expansion the air in the hot space has an average temperature lower than T_h. As this air leaves the hot space again it passes the heater, which is at the temperature

T_h: thus it passes through a finite temperature difference. The same is true for the cold space, *mutatis mutandis*.

The deviation from reversibility has the result that the actual efficiency is no longer equal to the theoretical efficiency given by equation (6). The effect is not large, however, (of the order of 10%), for two reasons. In the first place the decrease in temperature resulting from the adiabatic expansion is partly compensated by the fact that during the time of the whole expansion air at the temperature T_h still flows from the heater into the hot space. While it is true that this mixing of air of the temperature T_h with air of an average lower temperature is itself an irreversible process, it, nevertheless, reduces the magnitude of the effect, because the latter is determined entirely by the fall of temperature taking place. (Here again a quite analogous situation obtains in the cold space.)

The second reason why the effect is slight lies in the relatively low expansion ratio ($p_{\max} : p_{\min}$) which is employed in air engines. It lies between 2 and 2.5.

We shall not go here into the quantitative determination of the effect, for this leads to rather complicated calculations.

This discussion will now be concluded with a brief summing up of what goes on in the motor with respect to energy. For the sake of simplicity we again assume that in the hot space the temperature T_h always prevails and in the cold space T_c. For the isothermal expansion of the hot air heat is necessary, and this is supplied by the heater. In the regenerator the air is cooled, but the heat is stored and used again later for heating the air; it may therefore be left out of consideration in the balance of energy. In the expansion all the heat taken up is converted into mechanical energy. In the compression a part of it is used and again converted into heat, which is given off in the cooler. The net result of all the changes is that heat is continually being taken up from the heater by the work medium and partly converted into mechanical work and partly given off to the cooler.

The air process applied in a refrigerator

As already noted, the direction of the air cyclic process can be reversed. This means that it can also be made to take place in such a way that upon supplying a certain mechanical power $N = (1-\tau)\,Q$ per sec. an amount of heat τQ is taken from a reservoir at the low temperature T_c and an amount of heat Q is given off to a reservoir at the high temperature T_h. This reversion can be accomplished by choosing the phase difference φ already mentioned negative. Work is then not only taken up, as we have already noted, but the flow of heat also changes direction, *i.e.* heat is now removed from the cold space and given off to the hot space. This takes place with the same high efficiency that characterizes the air process in the engine. It is clear that the air process in this way functions as a refrigerator, and it has been found in practice, that this can in fact easily be realised. When the elements corresponding to heater, regenerator and cooler are given suitable dimensions very satisfactory refrigerators are obtained. In an experiment in which hydrogen was used as work medium a temperature of 80° K was attained in one stage.

CARRIER SUPPLY IN AN INSTALLATION FOR CARRIER TELEPHONY

by D. GOEDHART and G. HEPP.

621.395.44

In an installation for carrier telephony AC voltages of frequencies, which are whole multiples of 4 kc/sec, have to be supplied to a number of apparatus in the different telephone channels. The requirements made of these "carrier voltages" as to constancy of frequency and amplitude, freedom from distortion, etc. are explained in this article. For example the carrier frequencies for one and the same channel in the transmitting and in the receiving stations may differ by not more than about 1 c/sec. All the carrier frequencies can be excited collectively by means of circuits which give a highly distorted AC voltage, for example a periodic impulse with a fundamental frequency of 4 c/sec. This voltage also contains all the harmonics of 4 kc/sec, so that the desired carriers can be selected by a set of filters. The fundamental frequency of the impulse generator is kept contant by a "master oscillator", while the equality of the fundamental frequencies in the transmitting and in the receiving stations is ensured by synchronization of the two master oscillators by means of a transmitted synchronization signal. In the discussion of a carrier supply equipment designed by Philips a discussion is given of the circuits of the impulse generator, of the choice of impulse width, of the avoidance of frequency division in the impulse generator and of the manner of connecting the series of carrier filters with the impulse generator.

In ordinary long distance telephony the alternating currents from the microphone, which contain the speech frequencies of about 100 to 4000 c/sec, are amplified and transmitted directly over a pair of conductors of the cable between transmitting and receiving stations.

In carrier telephony the low-frequency speech vibrations are first modulated in the transmitting station, on a carrier of higher frequency, in much the same way as in radio broadcasting. By using a series of carriers with different frequencies, each of which "carries" a call (provides a telephone channel), a large number of calls can be transmitted simultaneously over each pair of conductors.

Systems have been developed with 4,12 or even more carriers (channels). A system designed by Philips before the war works with 17 channels. Compared with ordinary telephony, carrier telephony has the advantage that for a given number of simultaneous calls a much smaller number of pairs of conductors is needed, which means an appreciable saving of copper in the cables, although at the expense of a more complicated apparatus in the terminal stations.

The fundamentals of carrier telephony and several component parts of the apparatus employed were dealt with more or less extensively in this periodical in the years 1940-1942 [1]). Among the details which were not discussed at that time was the manner in which the necessary carriers are excited. This forms the subject of the present article. Other components of a carrier-telephony installation will be discussed in articles to be published in subsequent numbers of this periodical.

With respect to the examples to be discussed we shall keep to the apparatus already developed, to which the articles mentioned above apply [1]), *viz.* the 17 channel system with so-called single modulation. It has to be pointed out, however, that in the course of the past few years considerable developments have been achieved in the Philips laboratories with respect to carrier telephony. Systems for a large number of channels have been designed which differ considerably from those so far dealt with, both fundamentally and constructionally. The new developments, which have been made possible mainly because of the fact, that improved materials and component parts have meanwhile become available, will be dealt with in a new series of articles to follow the present ones.

For the sake of a better understanding of what is to be discussed here, it is desirable first to recall the most important features of a carrier-telephony installation. In *fig. 1* the main parts of the apparatus for two channels between the stations A and B are indicated. The microphone currents with the frequency q arriving from a subscriber A at station A are sent through a low-pass filter (LFZ), which, roughly speaking, has the task of suppressing the frequencies above 3400 c/sec, to the modulator

1) See for example Philips techn. Rev. **6**, 325, 1941 (Fundamentals); **7**, 83, 1941 (modulators); **7**, 104, 1942 (filters); **7**, 184, 1942 (equalization). (Volume 7 not yet published in English).

(*Mod.*), to which at the same time an AC voltage with the higher frequency p_1 (the carrier) is applied.

The modulator then delivers an output voltage which contains among other frequencies the "side-bands" $p_1 + q$ and $p_1 - q$ of the carrier. Following the modulator is a bandpass filter (*BFZ*), whose attenuation as a function of the frequency is of such a character that practically only one side-band is passed, thus only $p_1 + q$ if it is desired to use the higher side-band. The AC voltage thus obtained, which only contains components within the frequency band $p_1 + q$ (from about $p_1 + 300$ to about $p_1 + 3400$ c/sec), is combined with the similarly obtained voltages in the frequency bands $p_2 + q$, $p_3 + q$, . . . of calls of other subscribers, and after amplification put on a pair of conductors of the cable. At the other end of the cable, at the station *B*, a series of bandpass filters (*BFO*) is connected in parallel on the same pair of conductors, each of which filters passes only one of the frequency bands $p_1 + q$, $p_2 + q$, $p_3 + q$,... In this way the different channels are separated again. The voltages $p_1 + q$ of the call of subscriber *A* are passed by the corresponding band filter and applied to the demodulator (*Dem*). This apparatus is identical with the modulator at the transmitting end, and, as in that case, the carrier p_1 is here added. This results in a voltage at the output of the demodulator with the frequencies $p_1 - (p_1 + q) = = q$, which are the original low-frequency speech vibrations. In a final low-pass filter (*LFO*) these are freed of any undesired frequencies still present and finally passed on to the subscriber B_1.

The microphone currents from B_1 are transmitted to A_1 in exactly the same way over a different pair of conductors. As will be seen, a series of AC voltages p_1, p_2, p_3, . . . must be available at both stations for supplying the modulators and demodulators. The apparatus serving for the excitation of these carrier voltages must satisfy the following main conditions:

1. The frequencies of the carriers must be exactly the same at the transmitting station as at the receiving station, and must be extremely constant;
2. the carrier voltages must be free of harmonics and other undesired components;
3. the energy available must be sufficient for supplying the numerous modulators, demodulators and possible other apparatus [2]) which may be located at the station;
4. the amplitude of the carriers must not vary too much with time;
5. the apparatus must have the greatest possible reliability when in operation.

In the following we shall go deeper into these

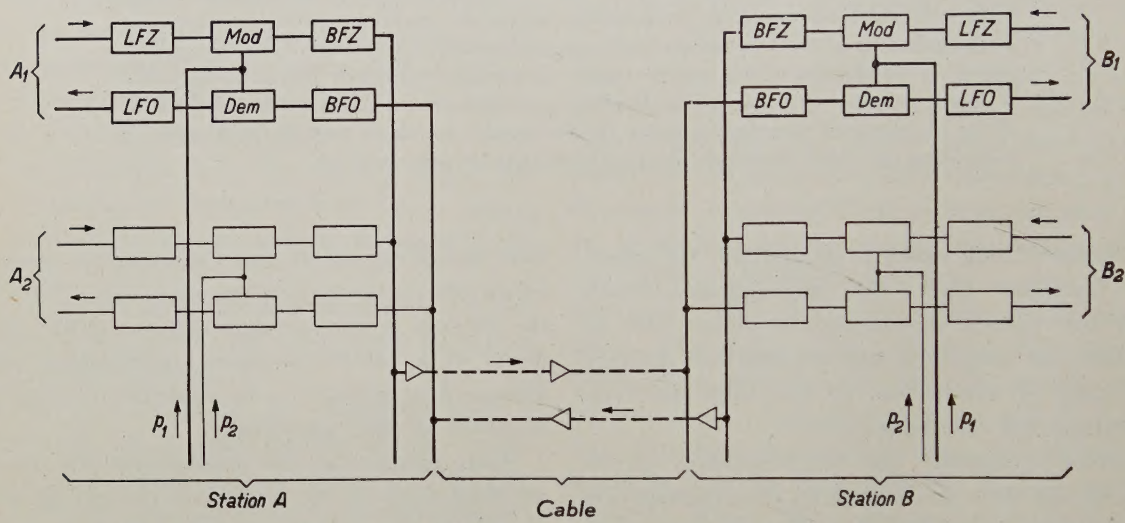

Fig. 1. Diagram of a carrier telephony connection between two stations *A* and *B*, drawn for two channels. The parts of one channel (with the carrier p_1) in which a call is taking place between the subcribers A_1 and B_1 are drawn with a heavy line. *LFZ* low-pass transmitting filter, *Mod* modulator, *BFZ* transmitting bandpass filter, *BFO* receiving bandpass filter, *Dem* demodulator, *LFO* low-pass receiving filter. At p_1, p_2, the carriers for the various channels are applied.

[2]) For example the carrier voltage in each channel can also be used for signalling, *i.e.* for the transmission of the dialling signals bringing about the connection with the desired subscriber.

conditions and explain how they are satisfied in practice. After that we shall discuss the most important parts of an apparatus constructed by Philips.

Stability of the carrier frequencies

Requirements

It is easy to understand why the carrier frequencies have to be exactly the same at the transmitting and receiving ends. Suppose that a call (frequencies q) is modulated in station A on a carrier p_1, but in station B it is demodulated with a slightly different carrier frequency $p_1 - \Delta p$. At the receiving end, instead of the original frequency q, the frequency $q + \Delta p$ is obtained: all the frequencies q of the speech spectrum are shifted by the same amount Δp. This shift in the spectrum is accompanied by a change in sound, which in the case of speech results in the first instance in a change in the individual voice so as to make it unrecognizable, while with larger shifts the speech even becomes unintelligible. Experiments have shown that a shift of 10 c/sec must already be considered as prohibitive. Music is even much more sensitive to a frequency shift: it looses its harmonic character and becomes dissonant [3]), a frequency shift of less than 1 c/sec already being disturbing. Since in carrier telephony installations the transmission of music has indeed often to be taken into account, for example for the connection of a broadcasting studio with a concert hall or with radio distribution centres, where channels with frequency bands two or three times as broad are reserved, the difference of about 1 c/sec is therefore the maximum difference permissible between the carriers in two stations.

This same high degree of accuracy is required both for the lowest and for the highest carrier frequencies employed. The necessary relative precision is therefore greatest in the case of the highest carriers; for example with a carrier of 68 kc/sec it amounts to about 10^{-3} percent. In order to comprehend what this means, imagine that this AC voltage were used for running electric clocks: the two clocks should then not differ by more than about one second per day.

Method of supplying the carriers

How then does one set to work in order to ensure this agreement between the carrier frequencies in two stations? It would be technically possible to use for the excitation of each carrier an oscillator whose frequency is kept extremely constant by some suitable construction (among other methods, by placing the frequency-determining element in a thermostat). It would then suffice to check the oscillators at regular intervals with a kind of "tuning fork" and to readjust them. Where we have a large number of channels, however, this becomes too laborious, so that other methods have had to be sought. According to recommendations of the C.C.I.F. (Comité consultatif international de téléphonie) whole multiples of 4 kc/sec should be chosen for the carrier frequencies. Thus for example in the 17-channel system mentioned the first 17 harmonics of 4 kc/sec were taken for the carriers: 4, 8, 12, . . . 60, 64, 68 kc/sec. Now, since a strongly distorted AC voltage with the fundamental frequency 4 kc/sec in general also contains all the harmonics thereof, it seems obvious to supply all the carriers mentioned collectively in each station by generating one such strongly deformed AC voltage and then separating the components by means of filters. In order to obtain constant carrier frequencies it is then only necessary to keep the fundamental frequency of the composite AC voltage sufficiently constant. This trouble, however, can also be saved, since in the first instance it is not actually a question of keeping the carrier frequencies absolutely constant, provided they are identical in the transmitting and receiving stations. If provision is made, by the transmission of a suitable synchronization signal, that the fundamental frequency of the distorted AC voltage is exactly the same in the two stations, then all the carriers of the two stations automatically correspond.

The apparatus which excites the AC voltage mentioned with all its harmonics will be dealt with further on. Here we would only observe that this generator of harmonics is kept at the correct fundamental frequency by a "master oscillator" tuned accurately to 4 kc/sec. From this generator the signal which is sent over the cable to synchronize the corresponding master oscillator in the other station is also derived. As synchronization signal the AC voltage of 4 kc/sec itself is not used, nor a harmonic of it which coincides with one of the other carriers, because owing to the modulation of these carriers with speech this would lead to interferences. It is preferable to use an unmodulated harmonic of 4 kc/sec that is higher than the highest carrier employed in the installation.

Summarizing we may represent the method

[3]) It may be expressed by saying that the ear has not an arithmetic, but a logarithmic perception of pitch. Thus upon transposing a piece of music all the frequencies should not be shifted by the same amount, but multiplied by the same factor. Experiments on the effect of frequency shifts have been described by J. F. Schouten, The perception of pitch, Philips techn. Rev. 5, 286, 1940.

outlined for the excitation of the carriers by a diagram as given in *fig. 2.*

When by means of the synchronization of the master oscillators the equality of the carriers in two stations has been ensured, a drift of the fundamental frequency is in itself not disturbing [4]). Nevertheless, in case the synchronization should fail, it is desired to be able at least to transmit speech intelligibly. Therefore measures are still taken to keep the frequency of the master oscillator in each station satisfactorily constant (see the thermostat in fig. 9), and it is regularly checked and readjusted so that even without synchronization the highest carrier in each station can never shift more than a few c/sec.

carrier voltage p, after being filtered out, is first fed to an amplifier capable of supplying the necessary power for all the modulators, demodulators, etc. of the channels working with p. In this amplifier distortion may occur to a larger or smaller degree, with the result that in addition to p, the harmonics $2p$, $3p$, etc. are also present in the output voltage. Finally there is the fact, that all the carrier filters, amplifiers, etc. are assembled in the same bay, so that if there is an undesired coupling in the apparatus one or more of the other carriers can be induced on the circuit for the carrier p.

Such a "contamination" of the carriers may lead to very undesirable disturbances. If a weak, unwanted voltage with the frequency P reaches

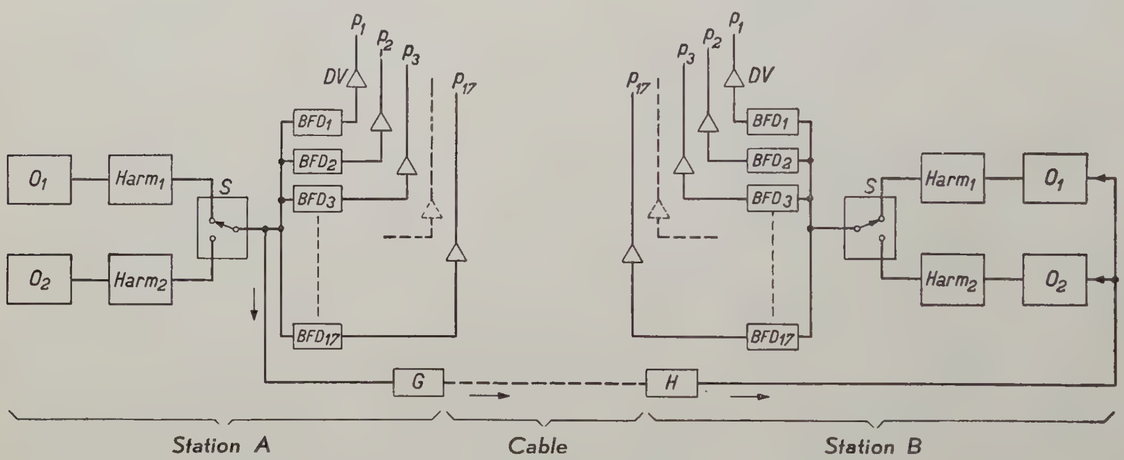

Fig. 2. Diagram of a carrier supply equipment, for instance for 17 channels at transmitting and receiving stations. O_1 master oscillator with reserve oscillator O_2, both tuned to 4 kc/sec; $Harm_1$ generator of harmonics with reserve generator $Harm_2$, $BFD_{1\text{-}17}$ carrier filters, DV carrier amplifiers; S automatic changeover; G filter for the synchronization signal, whose frequency lies above the highest carrier frequency used; H apparatus with which a frequency of 4 kc/sec is derived from the synchronization signals and fed to the oscillators O_1 and O_2.

Purity of the carrier frequencies

For various reasons the carrier voltage with the frequency p will never be quite purely sinusoidal. In the first place the carrier filters which have to select the different carriers (*BFD* in fig. 2), in addition to their "own" carrier, also transmit to a very small extent the adjacent carriers $p + 4$ and $p - 4$ and more distant ones. In the second place, in large stations from which many pairs of conductors go out, each with a complete carrier system, each

[4]) The condition also holds that the frequency spectrum may not be shifted too much with respect to the frequency characteristic of the band filters, since otherwise the highest or the lowest frequencies are more strongly attenuated than is permissible. But the tolerance in this case (about 30 c/sec shift) is still much greater than the one actually employed, which will be explained in the following.

the modulator in addition to the desired carrier p, upon modulation with the speech frequency q, the bands $P + q$ and $P - q$ will also occur in addition to the desired sideband $p + q$. If P is the adjacent carrier $p + 4$, $P + q = p + 4 + q$ is suppressed by the transmitting band filter. $P - q = p + 4 - q$ however, falls in the same frequency region as $p + q$.

This modulation product is thus passed in the normal way by the transmitting and receiving band pass filters of the channel and after demodulation with the carrier p gives rise to a frequency $4 - q$ at the receiving end. All the speech frequencies q of the channel are therefore transmitted in the same channel again with "inversion", at $4 - q$, which results in a disturbing unintelligible noise. With some systems a disturbance may arise when P is a carrier situated further away from p, that is to say in the event

that the transmitting bandpass-filters produce ample attenuation only immediately on either side of their band of transmission, with but little attenuation at greater frequency distances [5]). In such a case the modulation products $P + q$ and $P - q$ will be relatively little attenuated when they reach the cable. These frequencies are then passed at the receiving end by the receiving band filters of the channels with carrier P and $P - 4$ in the normal way, and after demodulation give the original speech frequency q respectively the inverted speech frequency $4 - q$ belonging to the channel p. While the latter frequency in the channel $P - 4$ again results in a disturbing unintelligible noise (unintelligible cross talk), the occurrence of q in the channel P, in addition to the frequencies Q of the call belonging to P, leads to an intelligible reproduction of the call intended for channel p. This "intelligible cross talk" is worse than the unintelligible sort, since it not only causes a disturbance of the call in channel P but also endangers the necessary secrecy of telephone calls.

Experience has shown that all these disturbing effects can be sufficiently restricted if care is taken that the admixtures with each carrier lie at least 60 db below the level of the carrier itself. This is obtained by the use of carrier filters, with sufficiently sharp cut off, by efficient shielding of the parts in the carrier supply bay and by the use of carrier amplifiers with extremely slight distortion. Amplifiers with strong back coupling are used.

Constant amplitude

If the amplitude of the carrier voltage on a modulator changes, and if the modulating voltage is small compared with the carrier voltage, the modulated voltage obtained will vary only slightly in intensity (in first approximation not at all). The fluctuations which occur normally in the carrier amplitude as a result of variations in mains or battery voltages or of the ageing of valves, can therefore be tolerated without difficulty as long as they do not exceed a value of for instance 10 percent. The effect upon the carrier amplitude of a variation in the loading of the carrier supply equipment — for instance when fewer channels are in use at night than in the daytime — can be limited sufficiently by keeping the internal impedance of the outputs of the carrier supply equipment small.

Reliability during operation

Serious consideration must be given to the possibility that a carrier voltage, or even the whole carrier supply may fail due to a disturbance. If one carrier fails, only the channel (or channels) working with that carrier is put out of operation. Provision must be made for an alarm signal to warn the operators immediately, so that the defect can be repaired. If, however, the whole carrier supply fails, the transmission on all the channels of all the pairs of conductors comes to a standstill. For such a calamity it is not enough to warn the operators. Therefore, in addition to all the precautions, which are taken to minimize the chance of such a disturbance, special circuits are provided which upon the failure of a vital piece of apparatus automatically and without any interruption switch over to a reserve apparatus. This holds in particular for the master oscillator and the generator of harmonics in the above apparatus (fig. 2). The only other reserve needed here is a single reserve amplifier for all carrier amplifiers. The fact that so little reserve apparatus is necessary is an additional advantage of the method of combined excitation of the carriers over the use of separate oscillators, in which a separate reserve oscillator would have to be provided for every wave length.

The practical construction of an apparatus

In the discussion of the construction of an apparatus for carrier supply we shall limit ourselves mainly to the most essential parts, namely the generator of harmonics and the filters.

Choice of the form of voltage to be excited

There are all kinds of distorted AC voltages containing all the harmonics of the fundamental frequency. The choice of the form of voltage to be excited however, is limited by the desire, that the different harmonics should not differ too much in amplitude. If one of the harmonics has a much stronger neighbour a very complex and expensive filter is needed for that harmonic in order to suppress this neighbour sufficiently. This holds especially for the highest harmonics where the neighbouring frequencies are relatively closest together.

A very suitable form of voltage is the periodic impulse, see *fig. 3*. If this is very narrow it contains all the harmonics in practically the same strength, or better, in the same weakness, because with impulses of short duration the power is, of course, only small. With a greater width of impulse the amplitude, at least that of the lower harmonics, increases proportionally, but at the same time with increas-

[5]) The attenuation necessary in these frequency regions in the channel p is provided by the low-pass filter *LFO*.

ing order of harmonics the amplitude shows a steadily more pronounced decrease [6]). If one considers especially the highest harmonics to be used as carrier, it will be seen that, under the influence of the two opposite effects they will at

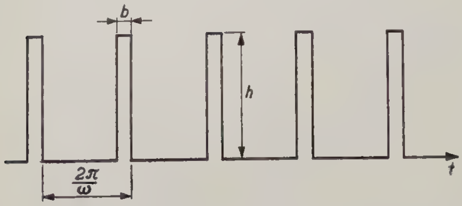

Fig. 3. Periodic impulse from whose harmonics the desired carrier frequencies can be filtered out. It is characterized by the period $(2\pi/\omega)$. the impulse width b and the impulse height h.

first become stronger and then weaker with increasing width of impulse (*fig. 4*). That width of impulse at which the amplitude of the highest harmonic assumes its maximum value is the most suitable one for our purpose, because in the first place that harmonic is in any case the weakest of the series, and in the second place it is most necessary for that harmonic to obtain a sufficiently large amplitude, since in the corresponding carrier filter the transmission region must be relatively the narrowest and therefore the attenuation in the transmission region is the strongest.

The above-mentioned optimum width of impulse can easily be calculated. If the height of the impulses is h and if a is the relative width of impulse (*i.e.* the duration b of an impulse divided by the period $2\pi/\omega$), then the variation of the voltage $f(t)$ of the periodic impulse can be given by the Fourier series:

$$\frac{\pi}{2h}\cdot f(t) = \frac{\pi a}{2} + \frac{\sin \pi a}{1}\cos \omega t + \frac{\sin 2\pi a}{2}\cos 2\omega t + \dots$$

If m is the order of the highest harmonic to be used, we want to choose a such that the amplitude $\sin m\pi a/m$ of this harmonic is a maximum. The condition for this is plainly that $\sin m\pi a = 1$, therefore $a = 1/_2\, m$.

It should be pointed out that with this width of impulse the desire for practically equal amplitudes for all the harmonics is also very satisfactorily fulfilled. For the highest harmonic the amplitude is $1/m$, for the lowest $\sin(\pi/2m) \approx \pi/2\,m$. The ratio between these amplitudes is $\pi/2$, *i.e.* the largest difference in level in the series of carriers amounts to only about 4db.

[6]) *Cf.* for example: J. F. Schouten, Synthetic sound, Philips techn. Rev. 4, 167, 1939.

Circuits of the impulse generator

All kinds of circuits can be used to excite periodic impulses. One of the simplest, the principle of which is also used in the Philips 17-channel system already mentioned, is shown in *fig. 5*. The circuit includes an amplifier valve V strongly back-coupled with the transformer T_1. Thus when anode current begins to flow through its primary winding the transformer gives the control grid of the valve a positive voltage. Due to the positive grid voltage the anode current becomes stronger, the grid becomes still more positive, the anode current increases still more, and so on. The resulting very rapid growth of the anode current may be compared with the breakdown in gas discharge. At a given moment the anode current can no longer

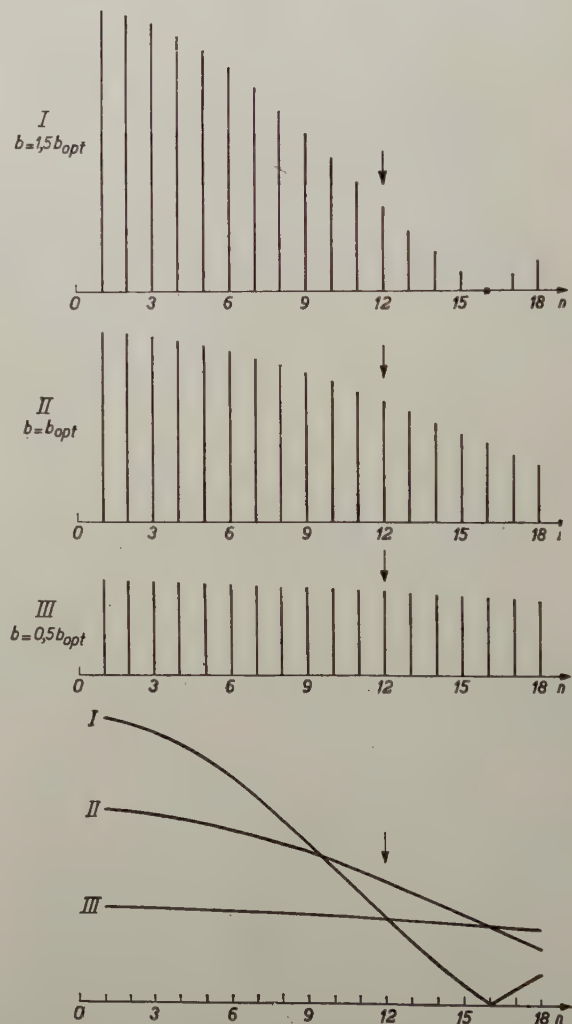

Fig. 4. Frequency spectrum of a periodic impulse with a given fundamental frequency and height. In the upper three figures the spectrum for three different values of the impulse width b is drawn, while in the lowest figure the envelopes of these three spectra are shown. It is clear that each harmonic assumes a maximum amplitude at a given impulse width. The optimum width for the 12th harmonic is here indicated by b_{opt}.

increase, since it is limited by the properties of the valve. When, however, the increase in the anode current ceases, the positive grid voltage delivered by the transformer fails, the anode current begins to fall and the transformer then delivers a negative

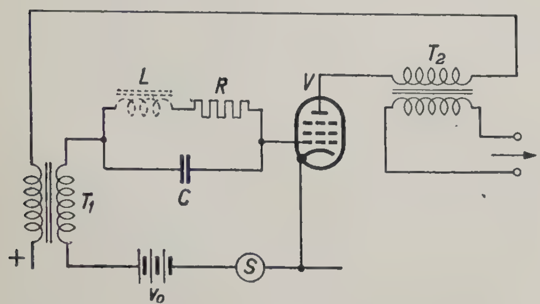

Fig. 5. Circuit for the excitation of periodic impulses. Due to the back-coupling *via* the transformer T_1, the amplifier valve V delivers an anode current in the form of short impulse. The "blocking" condenser C with the leakage resistance R provides that these impulses follow each other with approximately the correct period, while the period is accurately regulated by a synchronization voltage S. The periodic impulse obtained is supplied to an output valve *via* the transformer T_2.

voltage to the grid which accelerates the fall and in a short time causes the anode current to cease entirely. The anode current thus behaves as an impulse, whose width can be regulated for instance by a suitable choice of the transformer selfinductance.

Without the "blocking condenser" C and the leakage resistance R indicated in fig. 5 a second impulse would immediately follow the first one and so on. During the time that the grid was positive, however, the condenser was being charged by the grid current. After the interruption of the transformer voltage making the grid positive, the charge on the condenser causes a n e g a t i v e grid voltage which continues even after the end of the impulse and keeps the valve "blocked" until the greater part of the charge has been equalized over the leakage resistance. Then a new impulse follows, with a renewed charging of the condenser, this process repeating itself at a frequency mainly determined by the values of R and C. But the properties of the valve and the supplying voltage also play a part here. The fundamental frequency of the periodic impulse obtained in this way, which can be applied to the carrier filters *via* the output transformer I_2, is of itself not yet very constant. In order to obtain the required high degree of constancy, the impulse generator is synchronized by the above-mentioned master oscillator, which gives a very constant frequency of 4 kc/sec. For this purpose that oscillator voltage, together with a strong negative bias

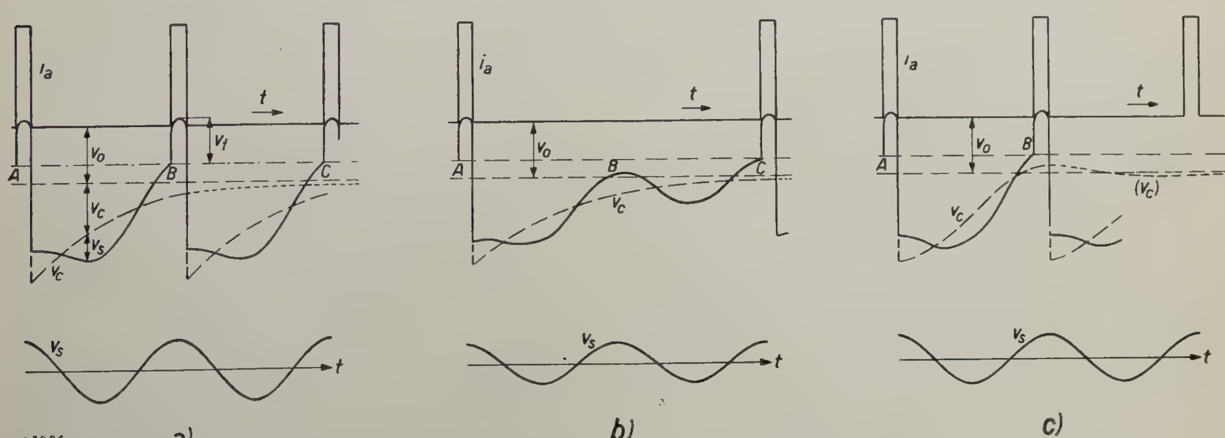

Fig. 6. *a*) The synchronization of the impulses by the sinusoidal oscillator voltage v_s drawn below. Below the time-axis (in the upper figure) the grid voltage of the valve V in fig. 5 is plotted, above the axis the anode current. The negative grid bias v_0 prevents the valve from "breaking down", even when the blocking voltage v_c of the blocking condenser has entirely disappeared. By the superposition of v_s on v_c, the bias v_0 is exceeded at each positive peak of v_s (for instance at B), and the valve is free to function. The dot-dash line indicates the level of the grid voltage at which the valve breaks down. v_t is the grid voltage contribution occurring at that moment from the transformer T_1, in fig. 5. The part of the curve v_c indicated by dots would only be traced if no break-down occurred.

b) The occurrence of frequency division. If v_s is not large enough, the breakdown at the desired moment B fails to happen and occurs only in the next (C) or a still later cycle of the synchronizing voltage.

c) The suppression of frequency division. By including the self-induction L, indicated by a dotted line, in the circuit of fig. 5 the "blocking" voltage v_c assumes the character shown here; at the moment B it does not block, but promotes the breakdown of the valve. If v_0 is so large or v_c so small that the moment B still passes without breakdown, the impulse generator fails entirely and the reserve generator goes into action.

is applied to the grid of the amplifier valve. The bias, prevents the valve from "breaking down" of itself; only when that voltage is compensated by the synchronizing voltage, thus at the positive peaks of the latter, can "breakdown" occur, so that it is forced into the rhythm of the synchronization voltage (see *fig. 6a*).

The avoidance of frequency division

A familiar phenomenon in such circuits is the occasional occurrence of frequency division. With insufficiently high synchronization voltage the breakdown at the prescribed moment sometimes fails to occur, owing to the fact, that the necessary level has not been reached, and it then occurs only in the next period, because on account of a further discharge of the blocking condenser the level has then risen somewhat (fig. 6b). In that case an impulse is obtained only once in two cycles of the synchronization voltage, and with still lower synchronization voltage sometimes only once in three, four or more cycles.

When the circuits are used for carrier supply this phenomenon is very undesirable. With frequency division of 1 to 2 the impulses would have a fundamental frequency of 2 kc/sec instead of 4 kc/sec, and the voltage would be too low for all carriers, but particularly the filters would be unable to suppress the extra harmonics lying at a distance of only 2000 c/sec away, so that a loud whistling tone would occur in all channels.

In order to prevent this, it is possible to introduce, in series with the leakage resistance R, a self-inductance L (shown with a dotted line in fig. 5) of such dimensions that "over discharge" (opposite charging) of the blocking condenser occurs at the moment, at which the following breakdown should occur. The "blocking" voltage thus changes its sign at that moment and promotes the breakdown. If in spite of this there is no breakdown, at the following positive peak of the synchronization voltage there will be even less chance of breakdown since then the "blocking" voltage, which has in the meantime fallen again, no longer helps, or at least only to a less extent (see fig. 6c). The valve thus remains blocked, *i.e.* the impulse generator fails entirely. This is much less serious than if it continued to function with frequency division, because upon its failure the reserve generator is brought into action by the automatic switching device, and the installation continues to operate without disturbance, while at the same time the operators are warned by an alarm signal of the defect in the generator thrown out of action.

Connection of the carrier filters to the impulse generator

In order to obtain the necessary carrier energy, the impulse voltage excited by the circuits already described is amplified by an output pentode in whose anode circuit the series of carrier filters is included. Each separate harmonic selected by one of the filters is then amplified again in the previously mentioned carrier amplifier and can then be applied as carrier to the various apparatus of the corresponding channel.

The connection of the filters to the output pentode mentioned will now be considered somewhat more closely.

The impulse voltage is applied to the control grid of the pentode in such a way that during each impulse the latter reaches zero potential, and in the intervals is strongly negative. A large anode current then flows through the loading impedance (the series of carrier filters) only during the impulses, while in the intervals the valve can deliver no current. This amounts to the same thing as if between the impulses the carrier filters were cut off by a switch from the source. A peculiar difficulty arises from this, which we shall attempt to explain. For the sake of simplicity we may consider the carrier filters as a series of single

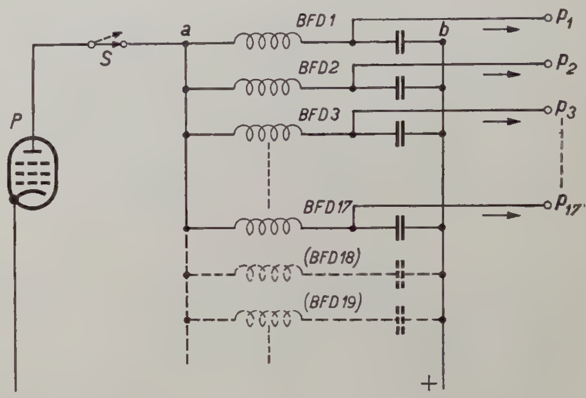

Fig. 7. Circuit of the series of carrier filters (BFD_1 to BFD_{17}), which are here represented simply as ordinary L-C-series circuits, with the output pentode P. The latter behaves as a source of current, which is switched on during the voltage impulses and off during the intervals by a switch S. The filtered carrier voltages are drawn off at p_1, p_2, . . .

L-C series circuits, tuned to 4, 8, 12, . . . kc/sec, each with a resonance resistance R and all connected in parallel, see *fig. 7*. One might also take a series of L-C parallel circuits connected in series, but in practice it is preferable, for instance because of the greater reliability, to work with elements connected in parallel, thus series circuits. (Actually of course each filter is very much more complicated than a single series circuit.) Now suppose that we wish to select only one carrier frequency of the impulse voltage, and thus include only one such series circuit in the anode circuit of the pentode. The action of that circuit then comes to nothing. A sinusoidal current should flow through the circuit, but during the greater part of the time, namely in the intervals between the impulses, no current at all can flow, since, as we have seen, the filter is then as it were part of an open circuit. If, however, we connect the whole series of filters in parallel in the anode circuit the situation is different. If the resonance resistance R

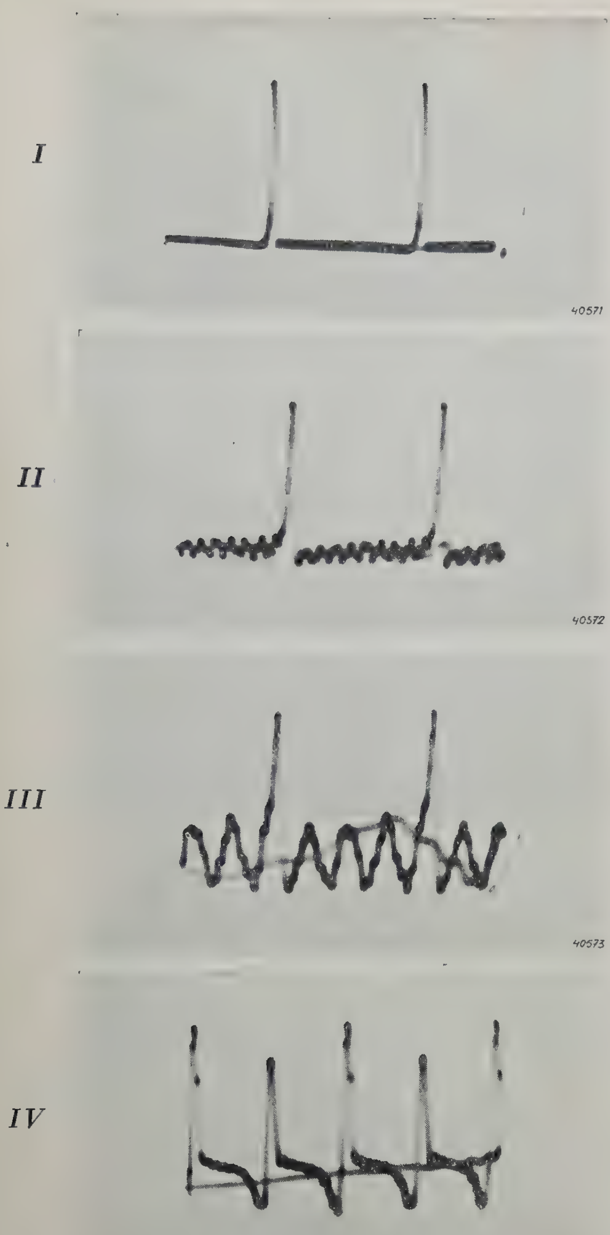

Fig. 8. Oscillograms of the voltage measured on a carrier supply bay between the points a and b in fig. 7.

I) The ideal case, obtained when there is a filter with very small resonance resistance R for all the harmonics contained in the impulse. (Recorded by replacing the whole series of filters by a resistance.)

II) The actual case in normal operation. The slight fluctuations in the time between the impulses indicate that for the higher harmonics the added extra filters and the correction network represent approximately, but not exactly, the desired low resistance R.

III) The carrier filter for the fourth harmonic has been removed. The remaining system of filters now has a much higher resistance for that harmonic, so that a considerable voltage with that frequency occurs between the points a and b. (This phenomenon can be used as a simple method of localizing any defect in one of the filters.)

IV) The carrier filters for all the even harmonics have been removed. Analogous to the case under (*III*) all the corresponding frequencies now occur in the voltage between a and b. Together these form another periodic impulse, but with a fundamental frequency twice as high as the one supplied to the output pentode. Thus a frequency doubling, as it were, has been realized by the remaining set of filters, when we take off the voltage between a and b.

is small enough so that practically all the current of a given frequency flows through the circuit tuned to that frequency, and if that current can be disregarded in all the other filters together, the system behaves as a resistance R for each of the harmonics and therefore for the whole impulse voltage. Thus we have a current in impulse form flowing through the connecting wires to the filters, *i.e.* in the time between two impulses the source need not supply any current, and it makes no difference whether or not the source is switched off from the filters in the time between two impulses.

The practical importance of this consideration is easy to understand: the desired action of the filters is ensured only when there is a filter present in the series for each of the harmonics occurring in the impulse voltage. Only then do all the currents of different frequencies flowing in the filters add together in such a way that in the interval between two impulses the total current is zero. If a number of filters are missing the other filters deliver less power of their own frequency. Thus even if we wish to use only a limited number of harmonics of the periodic impulse for carriers, we must have a filter for each of the remaining harmonics. In practice when all the harmonics up to a certain order are used, it is sufficient to take only a few more circuits than those necessary for the carrier frequencies, because, as appears from the Fourier series given above, the higher harmonics become rapidly weaker and thus have less effect; all the circuits for the very highest harmonics together can be finally replaced by a simple correction network.

The above can be demonstrated by means of a simple experiment. In a carrier supply bay of a 17-channel system with carriers from 4 to 68 kc/sec all the filters above 56 kc/sec are taken out of connection. The power of the carrier at the output of the 56 kc/sec filter thereby decreases no less than 50 percent.

It is interesting to note that the whole combination of filters behaves as a resonance circuit for an impulse voltage, in the same way as a single *L-C*-circuit behaves with respect to a sinusoidal voltage. In particular, when the attenuation constants (R/L) of all the filters are made equal, after the removal of the external impulse voltage the combination can be seen to oscillate with an impulse voltage, which decreases according to a power of e. The output voltage of the set of filters can also be used instead of the master oscillator to synchronize the impulse generator. The frequency of the latter is kept constant by the tuning of the filters in the same way as the sinusoidal vibration of an ordinary back-coupled oscillator is kept constant by an *L-C*-circuit in the anode circuit. Finally a "frequency multiplication" can even be realized, by removing all the filters of even order. A new set of filters is thus obtained, which forms as it were an "impulse resonance circuit" for double the fundamental

frequency of the impulse applied to it, and one then indeed obtains over the filters an impulse voltage with this double frequency. This surprising fact is illustrated by the last of the oscillograms in *fig. 8*. These oscillograms are explained in the text below the figure.

In order to obtain the greatest possible power out of the output pentode it is necessary here to make the resistance R of the filters equal to the internal resistance of the valve during the impulse. Actually, however, part of this optimum adaptation is sacrificed and R is made somewhat larger. The resultant decrease in power is scarcely worth mentioning, and the advantage is obtained that during the impulse the voltage drop in the valve, which is naturally small, since in a pentode a large anode current can already flow with a very small anode voltage, becomes still smaller, for example only 10 V, while the rest of the total feeding voltage, for instance 190 V, acts upon R. In this way an impulse height is obtained and with it an amplitude of the carriers which is affected practically only by fluctuations in the feeding voltage (mains voltage) and hardly at all by the properties of the valve or the magnitude of the control grid voltage. Thus the above mentioned requirement of constancy of the carrier amplitudes within 10 percent can easily be satisfied.

We give in *fig. 9* a photograph of the master-oscillator which determines the fundamental frequency of the whole series of carriers, and the corresponding reserve-oscillator, both mounted in the carrier supply bay of finished installation for carrier telephony. Some details are explained in the text below the figures.

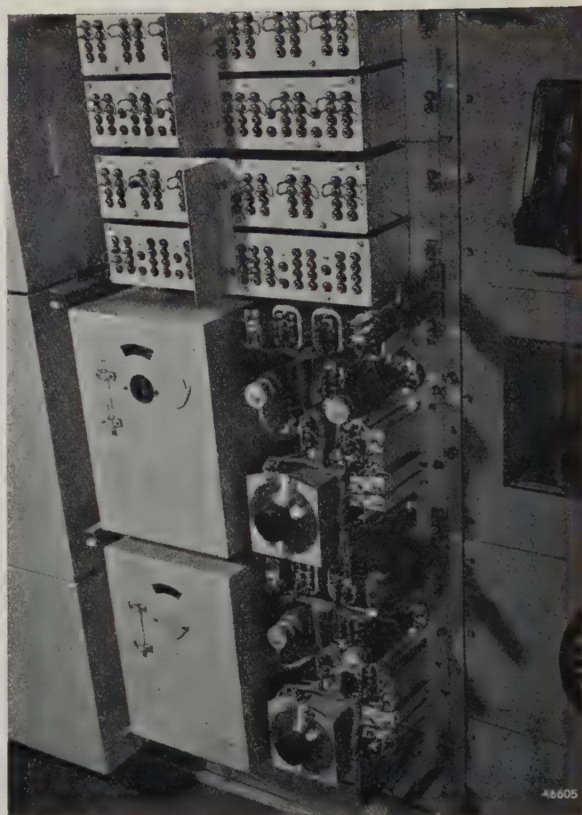

Fig. 9. The master oscillator with the (identical) reserve oscillator. In each oscillator in addition to the oscillator valve itself may be seen a second amplifier valve which serves as coupling valve, *i.e.* it prevents the oscillator valve being affected by the apparatus following it. The oscillation circuit is placed in a thermostat under the cover to the left. The temperature of this thermostat can be set by opening the window in the cover (in the upper oscillator it is open) and it can be read off on the thermometer beside it. With the rotating knob at the bottom of each oscillator the frequency can be very precisely adjusted (entire scale $\approx$ 1 c/sec).

MEASUREMENT OF THE ADHESIVE FORCE OF LACQUERS

by P. KOOLE.

620.179.4:620.198:667.7

The adhesive force of a lacquer is defined as the force necessary to scrape off a 1 cm wide strip of lacquer from an under-layer by means of a chisel. A description is given of a simple apparatus for measuring this force.

A layer of lacquer applied to a metal surface either to protect it from atmospheric influences or for esthetic reasons must, in order to fulfil its function, also have sufficient mechanical strength. This means that the lacquer must possess sufficient elasticity and hardness so that, when struck, it will not crumble or be scratched and that it must adhere firmly to the underlayer so as not to flake off or be easily abraded. The mechanical properties first mentioned, elasticity and hardness, can be measured fairly simply by the usual methods, but when judging the quality of a layer of a lacquer it is of no less importance to know the other property mentioned, the adhesive force of the lacquer, and there is as yet no generally accepted method of measuring this property. To begin with, there is no general agreement about what should be regarded as a measure of the adhesive power. One would be inclined to define the adhesive power as the force acting perpendicular to the surface of the lacquer which is necessary to remove 1 cm² of the lacquer film from its under-layer. In practice, however, this definition has no value, since an appropriate method of measuring such a force involves very great difficulties and, moreover, a lacquer film is in practice never subjected to such a treatment. The adhesive power can more appropriately be measured by the force which has to be exerted on a knife or chisel to scrape off, for example, a strip of lacquer 1 cm wide from its underlayer. It is of course true that in this force, besides the adhesive force proper determined by the nature of the lacquer and of the under-layer, the strength properties of the lacquer itself are discounted, and it depends therefore partially on the thickness of the layer of lacquer. Nevertheless, this force lends itself well to measurements on a large scale and corresponds somewhat to the forces occurring in practice which may cause mechanical injury to the lacquer film.

The simplest method of measurement based upon this definition consists in drawing a plate with a 1 cm wide lacquer film along a guide under a chisel slightly more than 1 cm wide and measuring the force which has to be exerted on the plate (see *fig. 1*). According to the sharpness of the chisel, a certain pressure on it is necessary in order to prevent it from sliding over the lacquer film. Due to this pressure, not only does a frictional force occur between the chisel and the under-layer — this could be determined, together with the frictional force between the plate and the guide, by repeating the experiment w i t h o u t the lacquer film — but at the same time the chisel has a tendency to penetrate into the base material, no matter how small the angle of inclination (α in fig. 1) is chosen.

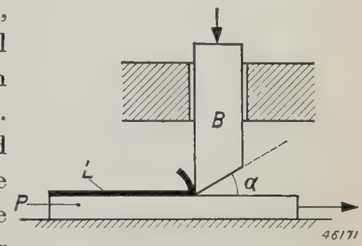

Fig. 1. The adhesive force of the lacquer film on the plate P can be measured by drawing the plate along a guide under the chisel B in such a way that the lacquer film is just scraped off the metal.

In order to avoid this, the chisel must be fixed rigidly and before each test the cutting edge must be so adjusted that it just shaves over the surface of the metal. The surface of the metal, however, must be perfectly smooth and flat, as otherwise the chisel will cut into the under-layer at some points and at others leave a thin layer of lacquer behind. It is therefore obvious that one cannot use any arbitrary lacquered plate for this test, but rather the lacquer film should be applied to a smooth under-layer specially prepared for the purpose. The requirements with regard to flatness of the under-layer prove to be so severe that in the set-up according to fig. 1 it was not even possible to grind the plate used as under-layer to a sufficienly accurate degree of flatness. We therefore decided to measure the adhesive force of the lacquer film on a c y l i n d r i c a l s u r f a c e, which it is possible to grind without any great difficulty to a degree of accuracy within 1 micron, which proved to be sufficient for our purpose.

With a cylindrical surface as under-layer it is, moreover, possible to apply a very simple method of operation. The relative motion between lacquer film and chisel can then be brought about by fixing the chisel to a lever rotating about the same axis as that of the cylinder with the lacquer film. In this way we arrived at the arrangement shown in *fig. 2*. The cylindrical surface is formed by the rim of a disc 1 cm thick, upon which the lacquer is applied with the aid of a spray stencil, the sides and a narrow strip of the cylindrical surface itself

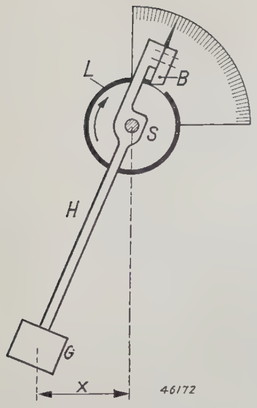

Fig. 2. Arrangement for the measurement of the adhesive force. *S* steel disc, accurately ground to circular shape, *L* lacquer film, *H* lever carrying the hard steel chisel *B*, *G* counter-weight.

being left bare. This bare strip serves as starting plane for the chisel, which is so adjusted that it just scrapes over the metal. The disc is now rotated slowly by means of a motor (at about 1 r.p.m.) in a direction opposite to the cutting direction of the chisel. The lacquer film tends to carry the chisel along with it, but to do so it must raise a counter-weight on the chisel lever. The largest couple that the lacquer film can exert on the lever is the product of the adhesive force and the radius of the disc. The couple of the counter-weight ($C \cdot x$) increases with increasing movement of the lever and at a given point the adhesive force is overcome and the lacquer is scraped off the disc. At this angle, which can be read off on a graduated arc, the lever remains in equilibrium during the rotation of the disc. The graduated arc can be calibrated directly in units of adhesive force (kg/cm).

Since the chisel also experiences a frictional resistance along the metal, and since a frictional force also occurs at the fulcrum of the lever, the experiment has to be repeated without the lacquer film on the disc. The difference between these two measured forces gives the adhesive force.

Fig. 3 is a photograph of an apparatus which has been in use for some time and has proved very satisfactory. With the regulatory resistance, which may be seen below, the number of revolutions of the motor can be adjusted to a suitable value. It may also be seen in the photograph that three different counter-weights can be used with the apparatus. This makes it possible to cover a wide range of measurements, since the adhesive forces of different kinds of lacquer vary considerably.

As has already been noted, the adhesive force according to our definition depends also on the thickness of the lacquer film. In order to obtain comparable values, therefore, it is necessary always to measure with the same thickness of lacquer, for instance 0.05 mm. The inevitable slight variations in thickness of the layer obtained by spraying on the lacquer — even for an experienced worker it will not be possible to keep to the thickness mentioned to within less than 0.01 mm — can in this set-up be eliminated very simply by applying a somewhat thicker layer and then, after it has dried, turning it to exactly the required thickness by means of the chisel. The necessary adjustment of the chisel is obtained by placing a metal strip of the required thickness under the chisel at the starting place.

The true adhesive force between lacquer and under-layer will not depend upon the thickness of the lacquer, but it will undoubtedly depend upon the nature of the surface of the under-layer. In our measurements, as has been seen, a smoothly ground under-layer must always be used, while in practice the surface to be covered will always be more or less rough.

It will sometimes even be roughened purposely, because the adhesion of the lacquer is then better. It is not *a priori* certain whether two kinds of lacquer, which adhere equally well to the smooth surface, will also adhere equally well to the rough surface. If, however, the adhesive force of a series of different lacquers is determined in the manner described, an order of evaluation is obtained which agrees perfectly with practical experience. Thus even although the relation between the adhesive force measured with the apparatus and the adhesion to be expected in practice is not entirely certain the values measured are quite useful as comparative figures for the quality of the lacquers to be used.

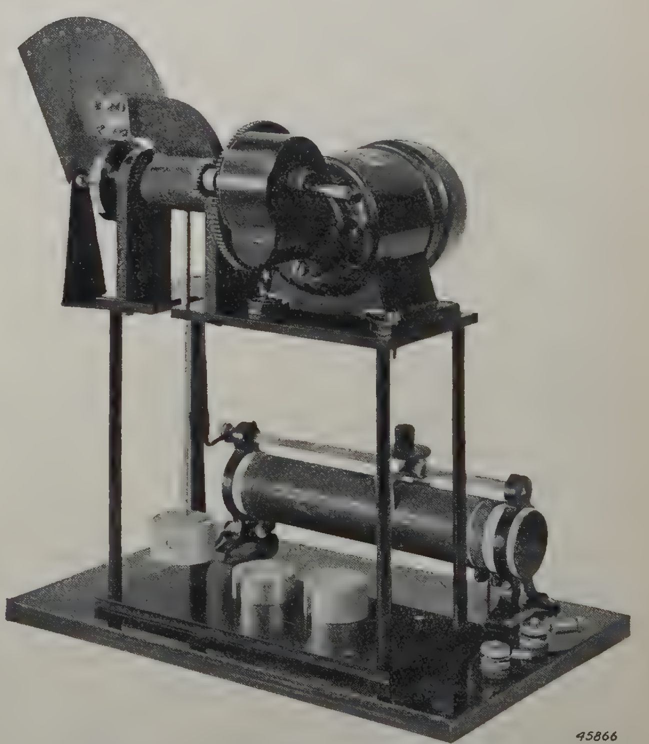

Fig. 3. Model of the adhesive-force meter. At the upper left may be seen the steel disc from which the lacquer film is scraped off, and the scale on which the necessary force is read off. At the upper right are the motor and the tooth-wheeled transmission for driving the disc. At the bottom a counter-weight may be seen suspended on the lever, and two counter-weights for different measuring ranges.

FLAT CAVITIES AS ELECTRICAL RESONATORS

by C. G. A. von LINDERN † and G. de VRIES. 538.565:538.566.5

After some introductory remarks about the characteristic vibrations which may occur in Lecher systems which are short-circuited at one end, this article deals with conical flat cavity resonators short-circuited around their outer edge and whose behaviour, as regards the rotation-symmetrical vibrations, corresponds entirely to that of the Lecher systems first mentioned. Subsequently the rotation-symmetrical characteristic vibrations of flat cavity resonators of more general forms are discussed and the variation of current and voltage with the radius is drawn for a flat cavity resonator. Resonance resistance and quality factor are calculated. Further it is indicated how the quality factor and the resonance resistance can be improved by making the cavity resonators thicker than those for which the theory given here applies unconditionally. In conclusion some examples are given of flat cavity resonators and their employment in high-frequency technique, for example for frequency stabilization and as output and input electrodes for short-wave transmitting valves.

In the excitation of high-frequency oscillations in radio transmitters, especially for very short waves, use is made of specially formed resonators, because at very high frequencies the ordinary oscillation circuits consisting of concentrated capacities and self-inductions possess too low a resonance resistance and too small a quality factor [1]). In addition to the so-called resonance cavities, Lecher systems consisting of two parallel conductors of uniform diameter as well as empty spaces with metal walls, which we call cavity resonators, are often used in high-frequency technology. In a Lecher system of a given length l stationary waves can be generated having a wave length of $4l$, provided the Lecher system is excited at one end by an electromagnetic oscillation, while the other end is short-circuited [2]). We shall now examine the free oscillations which may occur in cavity resonators of a general form, but in this article shall confine ourselves to flat cavity resonators, which have one dimension much smaller than the other two. When such flat cavity resonators are constructed in the form of solids of revolution, electromagnetic vibrations may occur in them which do not depend upon the angle of revolution, but only upon the radius. Such rotation-symmetrical oscillations thus actually depend upon only one coordinate. They can therefore still be treated in practically the same way as the oscillations of a Lecher system, which are of course also considered as depending exclusively on one coordinate, namely the length, since the transverse dimensions may be neglected compared with the length.

1) See for example our article "Resonance circuits for very high frequencies", Philips techn. Rev. **6**, 217, 1941, in which, *inter alia*, different equivalent definitions of the quality factor are explained in more detail.

2) The behaviour of a Lecher system with respect to travelling and stationary waves was discussed extensively by us in Philips techn. Rev. **6**, 241, 1941.

We shall first discuss the characteristic vibrations which may occur in a homogeneous Lecher system short-circuited at one end. Then we shall extend our discussion to include flat cavity resonators with double conic cross-section, after which we shall have something to say about the characteristic vibrations of flat cavity resonators of more general forms. After that we shall examine the influence of the resistance, giving values for resonance resistance and quality factor. These theoretical considerations will then be concluded with a discussion of the possibility of building up cavity resonators not having one dimension much smaller than the others, by piling up flat cavity resonators one upon another. Finally some examples are given of the practical use made of the cavity resonators here discussed in high-frequency technology.

Characteristic vibrations of Lecher systems

Let us consider a section of a Lecher system consisting of two parallel conductors of uniform cross-section and a given length l, which are connected at one end. We shall ignore the energy losses for the present, while for the capacity and self-induction per unit of length we introduce the symbols C^I and L^I. For the current i and the voltage V as functions of the time t and the coordinate of length x, the following equations hold:

$$\frac{\partial i}{\partial x} = -C^I \frac{\partial V}{\partial t} \text{ and } \frac{\partial V}{\partial x} = -L^I \frac{\partial i}{\partial t} \quad . \ . \ . \quad (1)$$

By differentiating these equations with respect to x and t four equations are obtained:

$$\left.\begin{aligned} \frac{\partial^2 i}{\partial x^2} &= -C^I \frac{\partial^2 V}{\partial x \partial t}; \quad & \frac{\partial^2 i}{\partial x \partial t} &= -C^I \frac{\partial^2 V}{\partial t^2}; \\ \frac{\partial^2 V}{\partial x^2} &= -L^I \frac{\partial^2 i}{\partial x \partial t}; \quad & \frac{\partial^2 V}{\partial x \partial t} &= -L^I \frac{\partial^2 i}{\partial t^2}, \end{aligned}\right\} \quad (2)$$

from which by combination two corresponding differential equations of the second order (the so-called vibration equations) for i and V can be derived:

$$\frac{\partial^2 i}{\partial x^2} = + L^I C^I \frac{\partial^2 i}{\partial t^2} \text{ and } \frac{\partial^2 V}{\partial x^2} = + L^I C^I \frac{\partial^2 V}{\partial t^2} \quad . \quad (3)$$

If now for the sake of simplicity we assign the coordinate $x = 0$ to the short-circuited end of the Lecher system (*fig. 1*) and to the open end $x = l$,

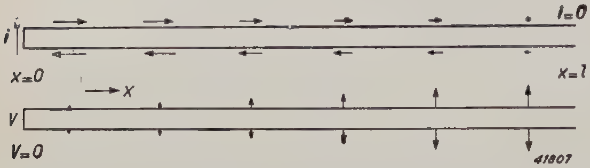

Fig. 1. Behaviour of current i and voltage V along a Lecher system short-circuited at $x = 0$ and open at $x = l$, while it vibrates at the fundamental frequency.

we require only those solutions of the vibration equations (3) for which the voltage V is zero for $x = 0$. For the characteristic vibrations of the short-circuited section of Lecher system there is the additional condition that at the open end $x = l$ the current i must be zero. The simplest stationary wave, which satisfies the vibration equations (3) and also these two conditions, is, except for a constant factor, the following [3]):

$$\left.\begin{aligned} i &= \sin \frac{\pi t}{2\,l\sqrt{L^I C^I}} \cos \frac{\pi x}{2l}, \\ V &= -\sqrt{\frac{L^I}{C^I}} \cos \frac{\pi t}{2\,l\sqrt{L^I C^I}} \sin \frac{\pi x}{2l}. \end{aligned}\right\} \quad . \; . \quad (4)$$

The variation of current and voltage along the Lecher system according to (4) is represented in fig. 1.

The coefficient of the time t in the expression (4) is the angular frequency $\omega = 2\pi\nu$. For the frequency ν_1 of the simplest characteristic vibration the following thus holds:

$$\nu_1 = \frac{1}{4\,l\sqrt{L^I C^I}}, \quad . \; . \; . \; . \; . \quad (5)$$

In the same way it may be seen from the expressions (4) that the length λ_1 of the largest stationary wave along the Lecher system short-circuited at one end is four times as great as the length l. The dependence of x on current and voltage can be represented by a cos or sine of $2\pi x/\lambda_1$, so that from (4) it does indeed follow that:

$$\lambda_1 = 4\,l \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

If (5) and (6) are multiplied by each other the well-known result follows that the product of the characteristic frequency ν_1 and the corresponding wave length λ_1 is equal to $1/\sqrt{L^I C^I}$, and this is the velocity of propagation v of the travelling waves which may occur along the Lecher system:

$$\nu_1 \lambda_1 = v = 1/\sqrt{L^I C^I} = \frac{3 \cdot 10^{10}}{\sqrt{\varepsilon\mu}} \text{ cm/sec.} \quad . \quad (7)$$

The expressions represented by (4) for current and voltage of the short-circuited Lecher system represent the so-called fundamental vibration of the latter. In addition to this, overtones also occur as characteristic vibrations of the same Lecher system (*fig. 2*), which also satisfy the vibration equations (3) and the conditions for voltage and current mentioned:

$$\left.\begin{aligned} i &= \sin \frac{(2k+1)\pi t}{2\,l\sqrt{L^I C^I}} \cos \frac{(2k+1)\pi x}{2l}, \\ V &= -\sqrt{\frac{L^I}{C^I}} \cos \frac{(2k+1)\pi t}{2\,l\sqrt{L^I C^I}} \sin \frac{(2k+1)\pi x}{2l}, \end{aligned}\right\} \; . \; . \quad (8)$$

where k may assume the values of the whole numbers. The characteristic frequencies ν_{2k+1} of these overtones lie harmonically and are the odd multiples of the fundamental frequency ν_1, namely:

$$\nu_{2k+1} = \frac{2k+1}{4\,l\sqrt{L^I C^I}} = (2k+1)\,\nu_1 \; . \; . \quad (9)$$

The corresponding wave lengths are:

$$\lambda_{2k+1} = \frac{4l}{2k+1} = \frac{\lambda_1}{2k+1}, \; . \; . \; . \quad (10)$$

so that the length l of the Lecher system is equal to an odd number of quarter wave lengths. In the case of homogeneous Lecher systems we are therefore only concerned with the familiar harmonic overtones which also occur with a vibrating string. This, however, is no longer the

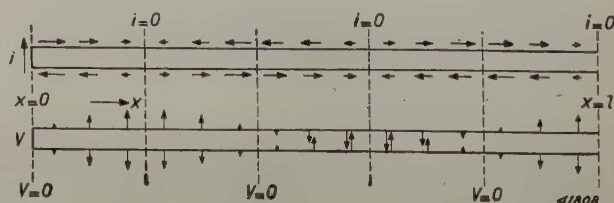

Fig. 2. Behaviour of current i and voltage V for the third characteristic vibration λ of a Lecher system short-circuited at one end. ($k = 2$ in equations (8), (9) and (10)), in the case of fundamental vibration.

[3]) The magnitude and sign of the coefficient in the expression for the voltage V is of course so chosen that the expressions (4) also satisfy the differential equations (1) of the first order for current and voltage from which we started in this discussion. $\sqrt{L^I/C^I}$ is often called the "wave resistance" (*cf.* Philips techn. Rev. **6**, 241, 1941).

case when the capacity and the self-induction depend upon the position, *i.e.* when the Lecher system is not homogeneous.

Then the position of the overtones is anharmonic, as we shall see later in this article.

Conical flat cavity resonators

If we were to imagine the homogeneous Lecher system short-circuited at one end, which we have considered until now, to be rotated so as to form a flat box, with or without a hole in it (*fig. 3*), we

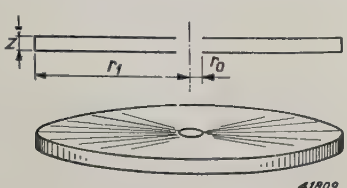

Fig. 3. Flat cavity resonator with thickness z and radius r_1 of the short-circuited outer edge. The radius of the hole is r_0. The flat cavety resonator originates by the revolution of a homogenious Lecher System, short-circuited at one end.

might expect that the rotatory-symmetrical characteristic vibrations of such a flat cavity resonator would exhibit a far-reaching similarity with those of a Lecher system short-circuited at one end. However, this does not give us the two-dimensional cavity resonator with the simplest characteristic vibrations, since in the vibration equations for current and voltage as functions of the radius r, the capacity and self-induction per ring of 1 cm width play a part in this rotatory-symmetrical case and these quantities, which we shall again call C^I and L^I, are not constant as in the homogeneous Lecher system, but depend upon the radius r of the ring being considered. It will be obvious that C^I is directly proportional to r, while L^I is inversely proportional to it; the expressions are as follows:

$$C^I = \frac{r}{2z} \text{ and } L^I = \frac{2z}{c^2 r}, \quad . \quad . \quad . \quad (11)$$

where z represents the thickness of the flat cavity resonator, the dielectric is a vacuum and c is the velocity of light.

A case, which is indeed mathematically entirely analogous to the homogeneous Lecher system, is a conical flat cavity resonator, for which the distance z between the two conductors is proportional to the radius r of the ring being considered (*fig. 4*). According to the ratios (11) C^I and L^I then do indeed again become independent of r. For the conical flat cavity resonator, therefore, we obtain exactly the same harmonic characteristic vibrations as for the homogeneous Lecher system, only in this case we are not dealing with a coordinate x which passes from $x = 0$ at the short-circuited

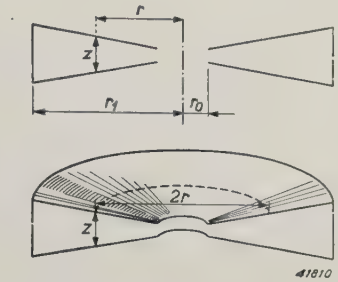

Fig. 4. Conical flat cavity resonator with thickness z at radius r. Outer radius r_1, inner radius r_0.

end to $x = l$ at the open end, but with a coordinate r which passes from $r = r_0$ at the open inner edge to $r = r_1$ at the short-circuited outer edge. Because of this the solutions of the vibration equations become somewhat more complicated than (4) and (8), but one is still dealing with an odd number of quarter sine waves of current and voltage (*fig. 5*), which now lie along the radius of the conical flat cavity resonator instead of along the length of the homogeneous Lecher system, so that in general the following is valid:

$$r_1 - r_0 = \frac{2k+1}{4}\, \lambda_{2k+1}, \quad . \quad . \quad . \quad . \quad (12)$$

which is quite analogous to relation (10) for the homogeneous Lecher system.

Characteristic vibrations of other flat cavity resonators

Flat cavity resonators whose thickness z depends in any arbitrary manner on r do not behave, as far as the electromagnetic characteristic vibrations are concerned, entirely like a Lecher system of uniform cross-section, as is the case for the conical flat cavity resonator just discussed.

In order to describe the axially symmetrical

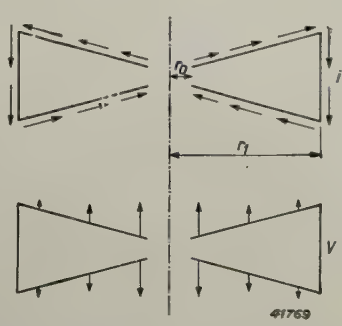

Fig. 5. Variation of current i and voltage V with the radius for a conical flat cavity resonator i: r_0 inner radius, r_1 outer radius.

characteristic vibrations which may occur in a flat cavity resonator with a given cross-section, we refer to the treatment of the characteristic vibrations of a so-called non-homogeneous Lecher system [4]), for which the cross-section and/or the distance between the two conductors depends upon the position (*fig. 6*). With Heaviside we shall now assume

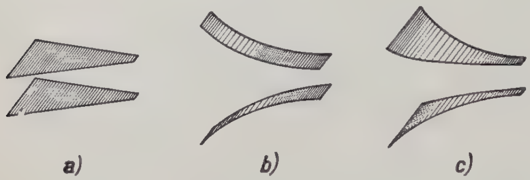

Fig. 6. Different forms of generalized Lecher systems consisting of:
a) two parallel strips of varying width,
b) two strips of uniform width at varying distance apart and
c) two strips which not only become wider but also diverge.

that the quantities C^I and L^I vary only slowly with the coordinate, which for the sake of simplicity we shall immediately call r, in order to obtain equations, which are also valid for the flat cavity resonators. We then obtain the following for the variations of current i and voltage V:

$$\frac{\partial i}{\partial r} = -C^I \frac{\partial V}{\partial t} \text{ and } \frac{\partial V}{\partial r} = -L^I \frac{\partial i}{\partial t} \quad . \quad . \quad (13)$$

These equations (13) are now again differentiated with respect to r and t:

$$\frac{\partial^2 i}{\partial r^2} = -\frac{\partial C^I}{\partial r}\frac{\partial V}{\partial t} - C^I \frac{\partial^2 V}{\partial r \partial t} \text{ and } \frac{\partial^2 i}{\partial r \partial t} = -C^I \frac{\partial^2 V}{\partial t^2};$$

$$\frac{\partial^2 V}{\partial r^2} = -\frac{\partial L^I}{\partial r}\frac{\partial i}{\partial t} - L^I \frac{\partial^2 i}{\partial r \partial t} \text{ and } \frac{\partial^2 V}{\partial r \partial t} = -L^I \frac{\partial^2 i}{\partial t^2}.$$

By combination of these equations, in which (13) has to be substituted for the first derivatives, about the same differential equations of the second order can again be found as vibration equations for i and V:

$$\left.\begin{aligned} \frac{\partial^2 i}{\partial r^2} - \frac{1}{C^I}\frac{\partial C^I}{\partial r}\frac{\partial i}{\partial r} - L^I C^I \frac{\partial^2 i}{\partial t^2} &= 0 \\ \frac{\partial^2 V}{\partial r^2} - \frac{1}{L^I}\frac{\partial L^I}{\partial r}\frac{\partial V}{\partial r} - L^I C^I \frac{\partial^2 V}{\partial t^2} &= 0 \end{aligned}\right\} \quad . \quad (14)$$

It may be seen that compared with the vibration equations (3) there is a middle term added due to the slow change of C^I and L^I with the coordinate r.

In order to obtain closed solutions of (14) we shall now assume as a special case that the thickness z of our flat cavity resonator is proportional to an arbitrary power n of the radius r (*fig. 7*):

$$z = h \left(\frac{r}{r_1}\right)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

According to (11) capacity and self-induction per ring 1 cm wide in a vacuum are:

$$C^I = \frac{r}{2h}\left(\frac{r_1}{r}\right)^n \text{ and } L^I = \frac{2h}{c^2 r}\left(\frac{r}{r_1}\right)^n, \quad . \quad . \quad (16)$$

thus, respectively, inversely and directly proportional to r^{n-1}. For this special case therefore the vibration equations (14) for current and voltage become:

$$\left.\begin{aligned} \frac{\partial^2 i}{\partial r^2} + \frac{n-1}{r}\frac{\partial i}{\partial r} - \frac{1}{c^2}\frac{\partial^2 i}{\partial t^2} &= 0, \\ \frac{\partial^2 V}{\partial r^2} + \frac{1-n}{r}\frac{\partial V}{\partial r} - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} &= 0. \end{aligned}\right\} \quad . \quad . \quad . \quad (17)$$

Just as for the simple vibration equations (3), we now look for solutions of (17) which are harmonic in the time t. For current i and voltage V we again take as variation with time sin and cos $2\pi ct/\lambda$, respectively. For the amplitudes of current (i_m) and voltage (V_m), which depend only upon the coordinate r, when we introduce $x = 2\pi r/\lambda$ as new independent variable we obtain the following equations:

$$\left.\begin{aligned} \frac{\partial^2 i_m}{\partial x^2} + \frac{n-1}{x}\frac{\partial i_m}{\partial x} + i_m &= 0, \\ \frac{\partial^2 V_m}{\partial x^2} + \frac{1-n}{x}\frac{\partial V_m}{\partial x} + V_m &= 0. \end{aligned}\right\} \quad . \quad . \quad (18)$$

For the case where $n = 1$ the middle term becomes equal to zero, so that the vibration equation (3) is again obtained for current and voltage. For $n = 1$ we have in fact the simple case, already discussed, of the conical flat cavity resonator with i and V varying harmonically in t and r.

Just as it is well known that the solution of (18) without the middle term are sines and cosines of

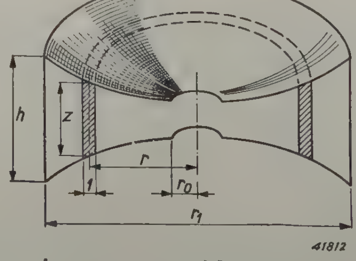

Fig. 7. Flat cavity resonator with thickness h at the outer edge, outer radius r_1 and inner radius r_0.

[4]) For a recent review of the theory of non-homogeneous Lecher systems see K. W. Wagner, Die Theorie ungleichförmiger Leitungen, Arch. Elektrotechn. **36**, 69, 1942.

x, it is also known that Bessel functions of the first and second sorts (J_p and N_p, where p denotes the "order" of the Bessel function) multiplied by powers of the coordinate x are solutions of (18). It is clear that not all solutions of (18) also satisfy the differential equations (13) with which we began. Those, for which this is indeed the case, are given by the following expressions:

$$\left.\begin{aligned} i_m &= x^{1-n/2}\,[A\,J_{n/2-1}(x) + B\,N_{n/2-1}(x)], \\ V_m &= -\sqrt{-1}\,\zeta\, x^{1-n/2}\,[AJ_{n/2}(x) + BN_{n/2}(x)]; \end{aligned}\right\}\quad (19)$$

A und B are arbitrary constants here and ζ is the wave resistance (*cf.* footnote [4]):

$$\zeta = \sqrt{\frac{L^I}{C^I}} = \frac{60\,h}{r_1^{\,n}}\,r^{n-1}\ \text{ohm}.$$

We must now choose A and B in (19) such that their expressions satisfy the boundary conditions.

In the first place the voltage must be zero at the short-circuited outer edge of the flat cavity resonator, *i.e.* $V_m = 0$ for $r = r_1$, or $x = 2\pi\, r_1/\lambda = b$. This boundary condition thus gives:

$$AJ_{n/2}(b) + BN_{n/2}(b) = 0 \quad . \quad . \quad . \quad (20)$$

By filling in the value of B from (20) in (19) we obtain:

$$i_m = Ax^{1-n/2}\,[N_{n/2}(b)\,J_{n/2-1}(x) - J_{n/2}(b)\,N_{n/2-1}(x)], \quad . \quad . \quad (21a)$$

$$V_m = -\sqrt{-1}\,A\,\zeta\, x^{1-n/2}\,[N_{n/2}(b)\,J_{n/2}(x) - J_{n/2}(b)\,N_{n/2}(x)] \quad . \quad . \quad (21b)$$

In the second place in the case of a flat cavity resonator with a hole in it (*i.e.* $r_0 > 0$) the current must be zero at the inner edge. This gives us the relation

$$N_{n/2}(b)\,J_{n/2-1}(a) - J_{n/2}(b)\,N_{n/2-1}(a) = 0, \quad (22)$$

where $a = 2\pi r_0/\lambda$. In the case of a flat cavity resonator without a hole (*i.e.* $r_0 = 0$), the current in the centre ($i = 0$) for $n > 0$ need not be zero because the top and bottom planes do not then touch each other. It certainly cannot, however, be infinite at that point, as would follow for $x = 0$ and $n \geqq 2$ from expression (21a).

It is found by closer inspection of this expression that the current remains finite for $x = 0$ and $n \geqq 2$ provided

$$J_{n/2}(b) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

For $n < 2$, except for $n = 1, -1, -3, \ldots$, the same condition (23) results in the fact that the current (21a) at the centre not only remains finite but is moreover equal to zero. In order to attain this for $n = 1, -1, -3, \ldots$, instead of (23), the condition must be fulfilled that

$$N_{n/2}(b) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Now since $a = 2\pi r_0 \nu/c$ and $b = 2\pi r_1 \nu/c$ and r_0, r_1 and n are fixed for a given flat cavity resonator, equation (22) (resonator with hole) or (23) or (24) (without hole) are only satisfied for certain values of the frequency ν. These special values of ν, which are thus the roots of equations (22) (23) or (24), respectively, represent the series of characteristic frequencies of the flat cavity resonator in question. The smallest positive root represents the fundamental tone.

It remains to be mentioned, that the values of the characteristic frequencies calculated from (22) pass over continuously into those calculated from (23) or (24) when a in (22), *i.e.* the radius of the hole, is allowed to approach zero.

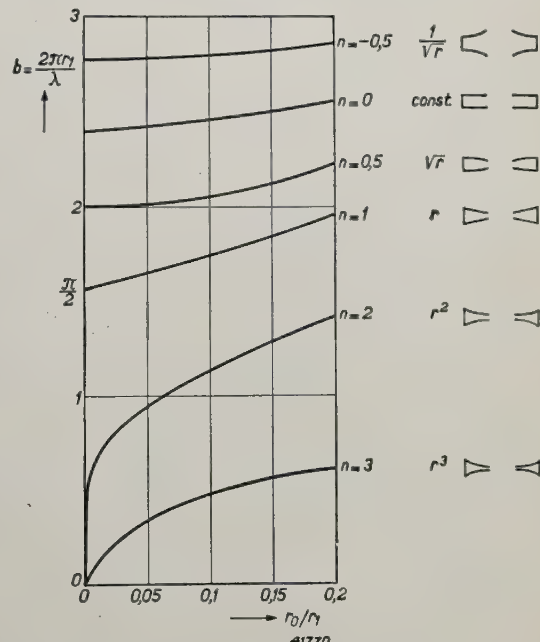

Fig. 8. The quantity $b = 2\pi r_1/\lambda$, which is proportional to the characteristic frequency, as a function of the ratio r_0/r_1 of the radii with different values of n for flat cavity resonators. For $n = 1$ and $r_0 = 0$ the quantity b becomes $\pi/2$, so that the wave length λ is then equal to four times the outer radius r_1.

In *fig. 8* it is now shown how at different values of n according to (22) the value of $2\pi r_1/\lambda$ for the fundamental tone varies with the quotient of inner and outer radii: $r_0/r_1 = a/b$. In the limiting case of a conical flat cavity resonator the wave length λ_1 of the fundamental tone is four times the difference between outer radius r_1 and inner radius r_0. It is evident from fig. 8 that this difference $r_1 - r_0$ is, however, in general by no means equal to a

quarter wave length. For instance in the case of a flat cavity resonator (fig. 3) with no hole ($n = 0$ and $r_0 = 0$), for which at the fundamental tone the following holds: $b = 2\pi r_1/\lambda_1 = 2.405$, the wave length becomes

$$\lambda = \frac{2\pi r_1}{2.405} = 2.61\ r_1.$$

If we now wish to know the wave lengths of several overtones for such a flat cavity resonator we can easily find them from the higher roots of equation (23) where n must be set equal to zero, so that we obtain:

$$J_0\,(2\pi r_1/\lambda) = 0\,. \quad . \quad . \quad . \quad . \quad . \quad (23')$$

The first three zero points of (J_0) lie respectively at $2\pi r_1/\lambda = 2.405$; 5.520; 8.653 (*cf. fig. 9*), from which then follow for the three longest wave lengths at which a flat cavity resonator with no hole can execute characteristic vibrations:

$$\lambda_1 = 2.61\ r_1;\ \lambda_2 = 1.14\ r_1;\ \lambda_3 = 0.73\ r_1\,. \quad (25)$$

The quotient of λ_2 and λ_3 is 1.56, which is not much smaller than $5/3 = 1.67$, which would be the ratio with a harmonic position of the characteristic frequencies. The frequencies of the higher overtones for the flat cavity resonator with no hole are indeed found to be progressively more nearly in the ratios of the successive odd numbers. Thus for example $\lambda_3/\lambda_4 = 1.36$ and $\lambda_4/\lambda_5 = 1.27$ while $7/5 = 1.40$ and $9/7 = 1.29$. It is only the ratio of the fundamental tone to the overtones which is far from harmonic, since $\lambda_1/\lambda_2 = 2.29$, which differs very much from 3!

In conclusion, for the simple case of the flat cavity resonator with no hole ($n = 0$; $r_1 = 0$) we shall examine the way in which the current and voltage are distributed along the radius on the basis of equations (21) and (23'). When we make use of the relation $J_{-1}\,(x) = -J_1\,(x)$ and omit the constant factors: we obtain the following:

$$\left.\begin{aligned} i_m &= x\ J_1\,(x)\,, \\ V_m &= J_0\,(x)\,. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

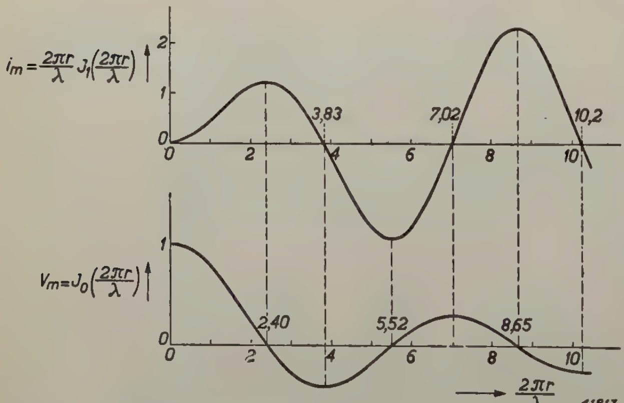

Fig. 9. Variation of the current amplitude i_m and the voltage amplitude V_m with the radius r of a flat cavity resonator. The behaviour from $x = 0$ to 2.405 gives current and voltage of the fundamental tone, from $x = 0$ to 5.520 the same for the second characteristic vibration, from $x = 0$ to 8.653 for the third characteristic vibration, and so on. In the same way at the zero points of the current curve xJ_1 lie the maxima for the voltage J_0.

The variation of current and voltage with the radius according to (26) is drawn in fig. 9. In the variation of the voltage we find the zero points of J_0 already mentioned in the derivation of the characteristic frequencies [5]), which are at the same time the points of the maxima of the current curve xJ_1.

The formulae given here lose their physical significance as soon as the absolute value of n becomes too large. The quantities C^I and L^I can then no longer be considered as varying slowly with r (*cf.* (16)), so that the assumption for which the initial equations (13) were derived is no longer satisfied.

Resonance resistance and quality factor

In the foregoing considerations we have assumed that we were concerned with flat cavity resonators having no ohmic resistance. Upon resonance no current then flows at the open inner edge, independent of the voltage acting on it, so that we would be dealing with an infinite resonance resistance. Now, however, we wish to calculate the resonance resistance and the quality factor which can be obtained in practice with flat cavity resonators, in which case, therefore, the energy losses should actually be taken into account.

The quality factor Q of a resonance circuit, except for a factor π, is the reciprocal of the logarithmic decrement and is thus a measure of the time in which a free vibration dies out in that circuit. The energy losses can now be taken into account by first determining the distribution of current and voltage as if there were no losses, and afterwards calculating the corresponding losses with this current and voltage distribution, without allowing them to influence that distribution. We shall follow the same method for the flat cavity resonators.

Like the self-induction L^I and the capacity C^I per cm width of ring (*cf.* fig. 7), the resistance R^I per cm width of ring now depends upon the radius r of the ring in question. It becomes

$$R^I = \frac{1}{2\pi r \sigma \delta}\,, \quad . \quad . \quad . \quad . \quad . \quad (27)$$

[5]) The short-circuited outer edge of the flat cavity resonator may of course be situated at each of these zero points.

where σ is the specific conductivity and δ the depth of penetration due to the skin effect for the frequency in question, which is given by the following expression:

$$\delta = \frac{1}{2\pi}\sqrt{\frac{c\lambda}{\sigma}}\,.$$

It is evident from this that for a conical flat cavity resonator also the resistance is by no means constant, but varies along the radius. The analogy with the homogeneous Lecher system thus indeed holds except for a factor. 2π, is itself the quotient of the field energy and the quality factor Q, the resonance resistance (in ohms) becomes :

$$Z = \frac{1}{2\pi}\cdot\frac{QV^2}{\text{field energy}} = 120\,Q\,\frac{h}{r_1}\,\mu_3, \quad . \quad (29)$$

where μ_3 again represents a numerical factor which is shown in *fig. 10c* for different values of n as a function of r_0/r_1.

In *fig. 11* for the case of a flat cavity resonator ($n = 0$) the quality factor Q is shown as a function

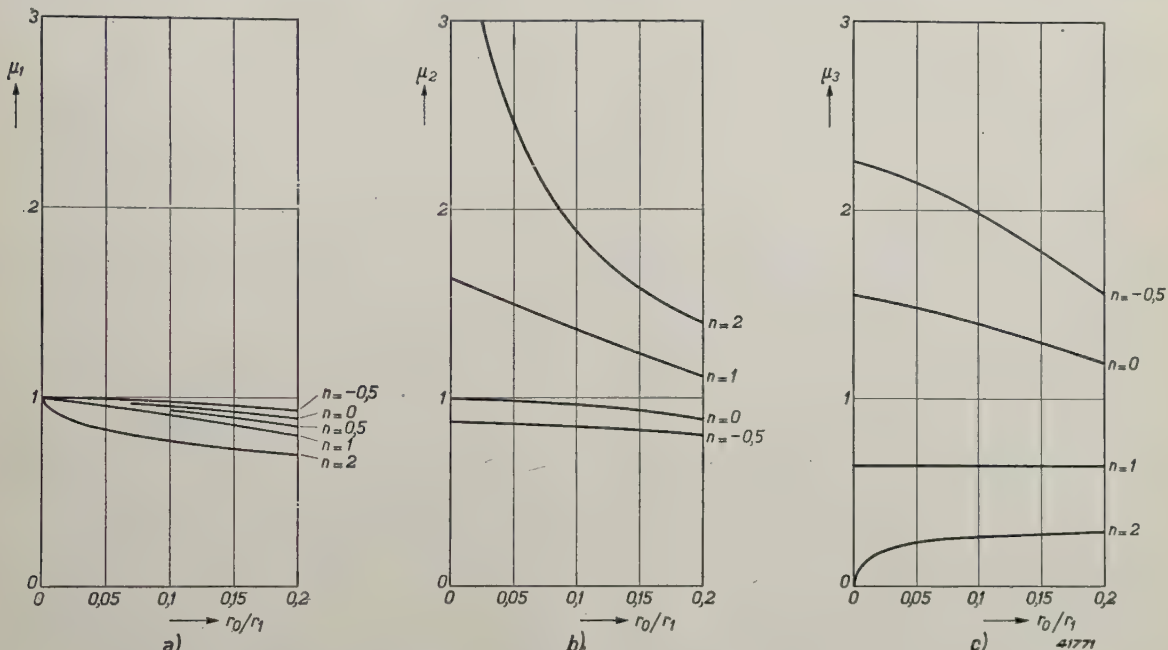

Fig. 10. The auxiliary quantities μ_1, μ_2 and μ_3 for different values of n as functions of r_0/r_1.

for the character of the vibrations and waves, but not for the energy consumption by the conical flat cavity resonator. Upon calculating the quality factor Q complicated formulae are obtained. For the fundamental vibration the quality factor can be written in the following form:

$$Q = \frac{h}{\delta}\,\frac{\mu_1}{\mu_2 + \dfrac{h}{r_1}}\,; \quad . \quad . \quad . \quad . \quad . \quad (28)$$

μ_1 and μ_2 here represent numerical factors which we have represented in *fig. 10a* and *b* for different values of n, as functions of r_0/r_1.

In the derivation of equation (28) we began with the fact that the quality factor Q is equal to 2π times the quotient of field energy and energy dissipated per period (see footnote [1])).

The resonance resistance of the flat cavity resonator may be considered as the quotient of the square of the effective voltage V for $r = r_0$ and the heat developed in it per second. Since the latter, of the ratio r_0/r_1 of the radii. The continuous (almost) straight line which is nearly horizontal is calculated according to equation (28). The small circles indicate the values determined experimentally. They

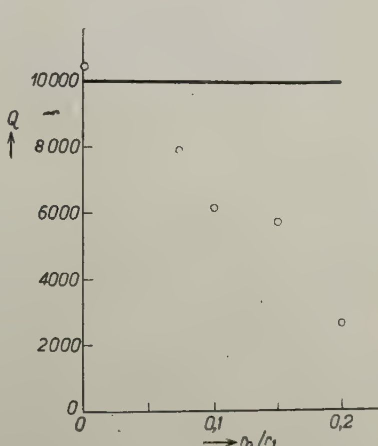

Fig. 11. The quality factor Q as a function of the ratio r_0/r_1 of the two radii of a flat cavity resonator ($n = 0$) with a thickness of 4 cm and an outer radius of 40 cm. For large holes it is found that smaller values of Q are measured than those calculated.

are derived from the resonance widths $\Delta\omega$ and the resonance frequencies ω_0: the quality factor may also be defined as $\omega_0/\Delta\omega$. It may be seen that for large holes the theory yields too large values of Q. This need not be surprising because in the preceding discussion the radiation losses were completely ignored.

The influence of the radiation losses, like that of the heat development, could now be taken into account beginning with the assumption that the radiation, like the ordinary ohmic resistance, does not change the established current distribution for a loss-free flat cavity resonator. It would, however, be necessary to take into account that the currents on the outside of the flat cavity resonator can no longer be disregarded as soon as the hole becomes larger. In the foregoing we have only considered the field inside the resonator. There the electric field is fairly homogeneous and has a vertical direction. Around the hole in the resonator, however, the lines of force are somewhat bent (*fig. 12a*) so that a small part of them even passes from the lower outside surface to the upper outside surface. There are therefore also charges on the outside of the resonator and their intensity varies at the same frequency as that at which the resonator vibrates. Consequently currents occur which flow on the outside but also continue on the inside (fig. 12*b*). In our calculation of the current distribution in the resonator, however, we assumed that $i = 0$ when $r = r_0$, so that a correction must actually be introduced here. This results not only in corrections of the characteristic frequency but also in an increase in the radiation losses. We shall not discuss this rather complex problem any further here.

Improvement of the quality factor and the resonance

As may be seen from formula (28) for the quality factor (Q), this quantity is in the first instance proportional to the ratio between the thickness of the flat cavity resonator and the depth of penetration δ due to the skin effect. Our previous considerations always referred to very thin cavity resonators for which $h \ll r$. If h is taken small enough to justify this treatment the quality factor will often be inadequate. This is not an insurmountable difficulty, since the cavity resonator can as required be made somewhat thicker and then be constructed as being built up of a number of thin resonators, for which the foregoing considerations are immediately valid. By such a piling up of flat cavity resonators, therefore, it is possible to deal also with thicker cavity resonators, without it being necessary to pass over to the general theory of cavity resonators of three dimensions.

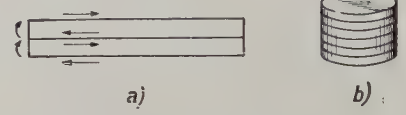

Fig. 13. By the stacking of thin cavity resonators (a) more general resonators can be built up (b).

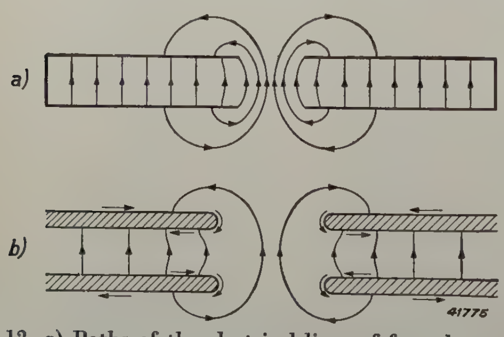

Fig. 12. a) Paths of the electrical lines of force between the plates of a flat cavity resonator. b) Electric currents induced in the vicinity of the hole as a result of the bending of the lines of force.

As is shown diagrammatically in *fig. 13*, for the case where $n = 0$ a cylindrical cavity resonator can very easily be obtained by the piling up of thin flat cavity resonators, the height of the final resonator no longer being small compared with the radius. In this cylinder, however, there are still partitions which are at the same time the top surface of one of the thin cavities and the bottom surface of the next. Since all these flat cavity resonators vibrate with the same characteristic frequency and all possess an equally intense vertical electric field, the situation is now simply that the currents belonging to the upper and lower cavities in such a partition are everywhere equal and opposite, so that the partitions could be removed from the cylindrical cavity resonator. One point will, however, then be altered; the heat development, which the electric currents in the partitions of the piled up thin cavities would produce, no longer occurs! This reduction of the losses by the removal of partitions is not merely an imaginary experiment but a true fact: as long as the partitions still remain in our cylindrical cavity resonator the heat losses will occur in it, because as a result of the skin effect the currents then actually flow only in the top and bottom layers of such a partition, so that they will by no means cancel each other as far as the development of heat is concerned.

Thus by the removal of the partitions we reduce the energy losses due to heat development compared with the total vibration energy, and it is therefore understandable that for such a more general cavity

resonator obtained by "stacking" we can obtain better values for the quality factor Q and the impedance Z than were possessed by the flat cavity resonator with which we began. If we calculate Q for a cylinder of height h and radius r, we obtain

$$Q = \frac{h}{\delta} \cdot \frac{h}{r_1 + h} \quad . \; . \; . \; . \; . \quad (30)$$

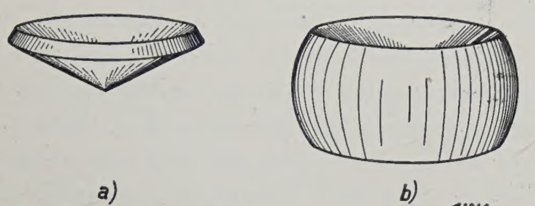

Fig. 14. Thin cavity resonators in the shape of conical shells can also be stacked.

In a similar way, by the stacking of the thin cavity resonators shown in *fig. 14a* in the form of conical shells, we can obtain the cavity resonator shown in fig. 14*b*, which has become very important in short-wave technology. In the discussion of several practical examples we shall also encounter such a cavity resonator (see fig. 20).

Several practical examples

For a copper conical flat cavity resonator (*fig. 15*) with $n = 1$, a thickness $h = 10$ cm at the outside edge and radii $r_0 = 0.6$ cm and $r_1 = 30$ cm, at a wave length of 122 cm, we measured a quality factor $Q = 10\,200$, while the value 11 400 was calculated. For the impedance Z a value of 273 600 ohms was calculated. If in this flat cavity resonator a power of 25 watts is developed in the form of heat, the current at the outer edge is $i = 130$ A, the voltage on the inner edge of the cavity amounts to $V_{r0} = 2600$ V, while the magnetic field strengths at the inner and outer edges are $H_{r0} = 1.4$ gauss and $H_{r1} = 0.9$ gauss. It was actually found that a small transmitter which could deliver a power of 25 W gave a current of about 100 A in the short-circuit ring, which could be concluded from the voltage generated in a loop.

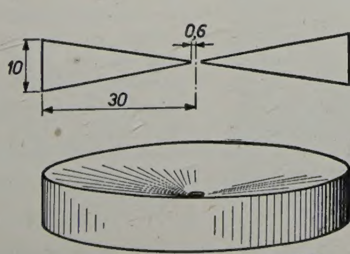

Fig. 15. Conical flat cavity resonator constructed with $n = 1$. The dimensions are given in centimeters.

For the case of a flat cavity resonator ($n = 0$) with a thickness $h = 4$ cm and an outer radius $r_1 = 40$ cm and different radii of the hole: $r_0 = 0$; 3; 4 cm, measurements gave the following values of the quality factor: $Q = 10\,350$; 7 700; 6 100.

Frequency stabilization with flat cavity resonators

Since a satisfactory quality factor Q can be obtained with flat cavity resonators, they can very well be used for keeping the frequencies of oscillators constant. The cavity resonator is then loosely coupled with the oscillator, so that in certain frequency regions the wave length is determined much more by the tuning of the resonator than by the rest of the connections joined to it. In *fig. 16* a conical flat cavity resonator is shown coupled with a Lecher system of variable length l_1. In *fig. 17* the

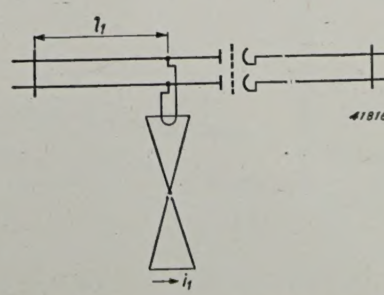

Fig. 16. A conical flat cavity resonator is coupled with a Lecher system of variable length l_1, in order to keep the frequency of the oscillator constant. In the photograph the loop by means of which the current i_1 is measured can just be seen on the upper right-hand edge of the flat resonator.

wave length λ at which this system vibrates and the amplitude i_1 of the current flowing in the short-circuit ring of the resonator are plotted as functions of

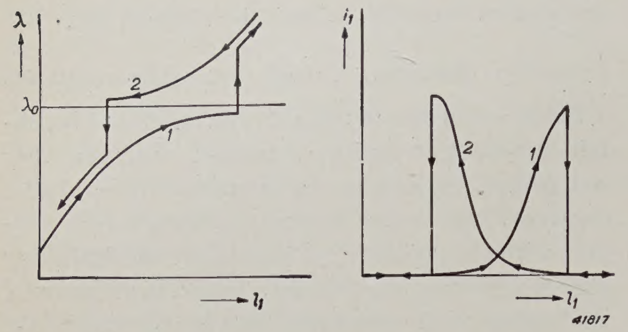

Fig. 17. The wave length λ and the current i_1 occurring with the connections shown in fig. 16 as functions of the length l_1 of the Lecher system. The figures represent a portion of the λ-l_1 plane and the i_1-l_1 plane, respectively, the points $\lambda = l_1 = 0$ and $i_1 = l_1 = 0$ falling outside the figures. Over a wide range the influence of l_1 on the wave length is found to be only slight, since the latter is mainly determined by the wave length λ_0 of the conical flat resonator. It is further evident from the figure that the state of oscillation is not determined unambiguously by the length l_1, but that it also depends upon the way in which a given tuning is reached.

the length l_1 of the Lecher system. Over a wide range the length l_1 of the Lecher system is found to have only very little effect on the wave length λ at which the whole system vibrates. The latter wave length is then determined almost exclusively by the wave length λ_0 of the free vibration of the conical flat cavity resonator. In this figure the disadvantage is also evident that the oscillator frequency is not wholly determined by the circuits alone but also depends upon the way in which the momentary state of the circuits was reached, so that the previous history of the state at a given moment also plays a part. This is a result of the non-linearity of the transmitting valves used [6]).

Cavity resonators as output and input electrodes with short-wave transmitting valves

When the high-frequency oscillation energy is taken from radio valves by means of the anode upon which the electrons must themselves finally impinge with high velocities, in the case of high-power valves a large amount of heat is developed in the anode. In the construction of such valves this should be taken into account, be it at some cost of considerations connected with high frequency. For this reason transmitting valves for very short wave lengths are at present often constructed so that the electrons pass along a pair of rings in which they induce charges. They then give off high-frequency oscillation energy to the rings without themselves striking the rings. The collision energy, however, is taken up by the anode, which is earthed for high frequency and may thus have any desired large dimensions.

In *fig. 18* such a so-called induction tube oscillator is shown. A beam of electrons whose intensity is controlled in a high-frequency rhythm (*fig. 19*) by a grid (*hf*) passes through a slit in a flat cavity

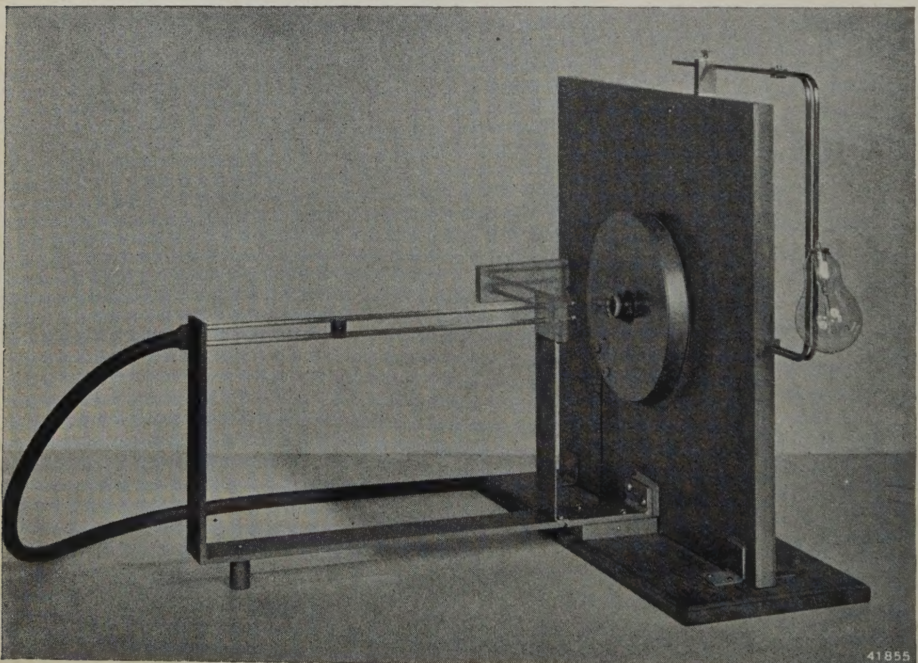

Fig. 18. Induction tube oscillator for waves of 65 cm, equipped with a vertically placed, rectangular flat cavity resonator for taking off the high-frequency oscillation energy. On the left-hand side of the flat resonator may be seen one of the magnetic coils which serve to keep the electron beam concentrated. The Lecher system extending to the left serves for tuning the grid circuit, while the Lecher system extending toward the rear conducts part of the oscillation energy excited in the flat resonator back to the grid, so that the electrical oscillations are maintained. The Lecher system to the right takes from the resonator the energy dissipated in the lamp serving as loading impedance.

6) *Cf.* for example Balth. van der Pol, Trillingshysteresis bij de triodegenerator met twee graden van vrijheid (Vibration hysteresis in the triode generator with two degrees of freedom). T. Ned. Radio Gen. 1, 125, 1921.

resonator (S), upon which electric charges are induced, while the electrons of the beam themselves finally impinge on the anode (a). Seen from the point of view of the cavity resonator, the induction tube seems to possess a fairly high adaptation resistance. In order to ensure a good transmission

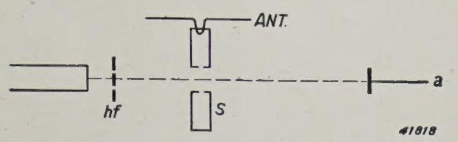

Fig. 19. Diagrammatic representation of an induction tube amplifier with a flat cavity resonator S as output electrode. The electrons are controlled by the high-frequency grid hf and move toward the anode a. The aerial (ANT) is coupled to the resonator, which is excited by the pulsating electron current passing through it.

of the high-frequency energy to the aerial, the impedance of the unloaded oscillation circuit must be high compared with the adaptation resistance. Therefore a cavity resonator with its large impedance is especially suitable to play the part of oscillator circuit here. The high-frequency energy is thereby taken from the cavity resonator by coupling the aerial to it with the help of a loop.

The resonator then functions mainly as transformer between the high-frequency electron beam supplying the energy and the aerial radiating it.

If it is impossible to obtain an adequate quality factor and resonance resistance with the thin, flat cavity resonator shown in figs. 18 and 19 in the induction tube oscillator, these quantities cannot be easily increased by using a somewhat thicker, flat resonator, since in order to keep the electron beam in the correct direction magnetic coils are situated along it at fairly short intervals and a resonator has to be fitted between the coils. It is, however, possible to obtain a larger quality factor and a higher resonance resistance by using a resonator like the one shown in fig. 14*b* whose thickness at the centre remains sufficiently small while at distances farther away from the electron beam the resonator becomes much thicker. In *fig. 20* an induction tube amplifier with such a thicker resonator is shown.

For the generation of high powers at very high frequencies much use is being made at present of a control mechanism with which the velocities of the electrons allowed to pass are varied and not the number. In their further progress the faster electrons will then overtake the slower ones, so that accumulations occur whereby the same effect is attained as with amplitude control. In the application of such so-called velocity modulator valves different cavity resonators must be introduced around the

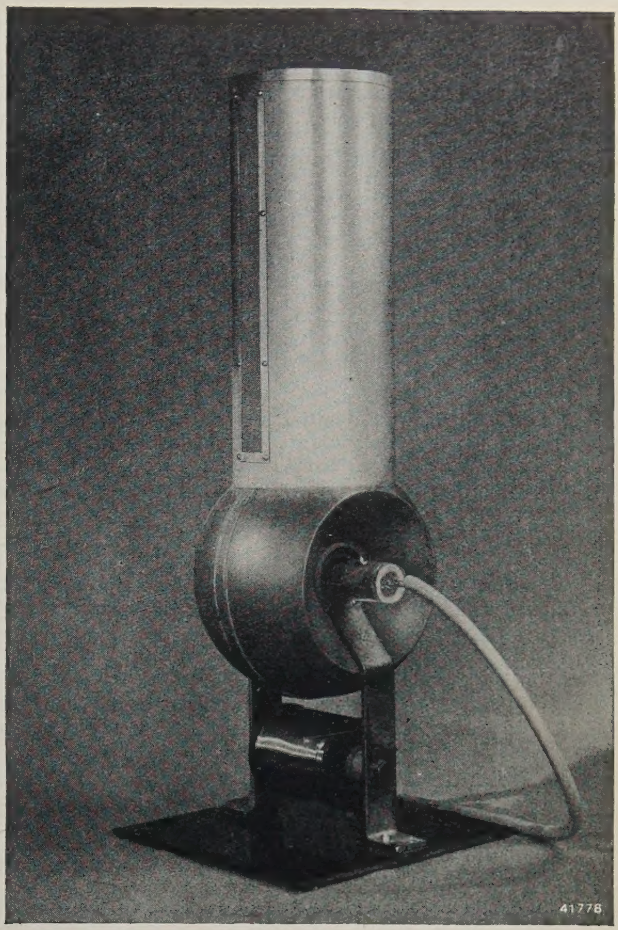

Fig. 20. Induction tube amplifier for 45 cm waves, equipped with a thicker cavity resonator like that shown in fig. 14*b* with which a larger quality factor and higher resonance resistance can be obtained than is possible with thinner resonators. In the cylinder on top of the cavity resonator is an artificial aerial in the form of an incandescent lamp. The coil below the resonator provides the magnetic field which keeps the electron beam concentrated.

path followed by the electrons from cathode to anode. Therefore thin, flat cavity resonators are often used here.

In *fig. 21* a velocity modulator amplifier is shown diagrammatically. In this case instead of being controlled by the grid hf of fig. 19 the electron beam is controlled by the slit of the input resonator S_1. Some distance farther along the path of the electrons is the output resonator S_2, so that

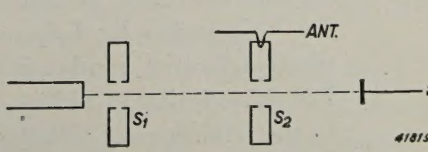

Fig. 21. Diagrammatic representation of a velocity modulating valve amplifier with flat input and output cavity resonators: S_1 and S_2. The electrons, on their path to the anode a, are controlled in a high-frequency rhythm by S_1, while then in passing S_2 they excite it in a high-frequency rhythm. The latter resonator passes on its oscillation energy to the aerial (ANT).

the electrons will arrive there with a variation in density sufficient to excite oscillations in S_2 with reasonable efficiency. The electrons themselves finally reach the anode a, while the flat resonator S_2 gives off its high-frequency oscillation energy to the aerial coupled with it.

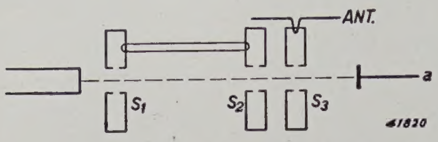

Fig. 22. The flat input and output cavity resonators S_1 and S_2 are coupled to each other in order to make it possible for the system to oscillate independently. The high-frequency oscillation energy for the aerial (ANT) is obtained from the pulsating electron current on its way toward the anode a by means a third flat cavity resonator S_3.

If the flat input and output resonators (S_1 and S_2) are coupled with each other, this system can be made to oscillate independently (*fig. 22*). The high-frequency oscillation energy is then finally taken from this velocity modulator oscillator with the help of a third flat resonator S_3 coupled with the aerial. A demonstration model of such a generator for very short waves is shown in *fig. 23*.

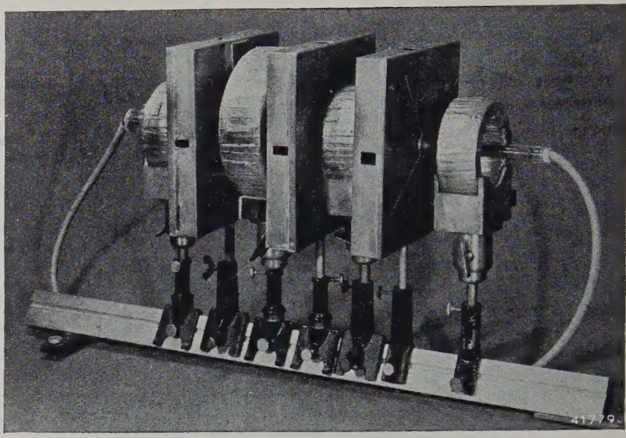

Fig. 23. Velocity modulator for waves of 40 cm in which use is made of three flat cavity resonators (*cf.* diagram of fig. 22). The cathode is on the left and the anode on the right. The size of the opening in the cavity resonator at the right S_3 can be regulated by turning the spokes visible in the photograph in order to change the characteristic frequency. The magnetic coils serve to keep the electron beam concentrated.

The last number of *Philips Research Reports* (No. 3 of volume 1, April 1946) contains following articles:

R12: K. F. Niessen: On the error in the determination of the median plane of a radiobeacon in a tilted airplane.

R13: B. D. H. Tellegen: Network synthesis, especially the synthesis of resistanceless four-terminal networks.

R14: H. B. G. Casimir: On Onsager's principle of microscopic reversibility.

R15: M. Gevers: The relation between the power factor and the temperature coefficient of the dielectric constant of solid dielectrics.

R16: T. Jurriaanse, F. M. Penning and J. H. A. Moubis: The cathode fall for molybdenum and zirconium in the rare gases.

R17: G. W. Rathenau and J. L. Snoek: Apparatus for measuring magnetic moments.

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